Lecture 9

Proof by Exhaustion (contd.), Existence Proof, Forward & Backward Reasoning

More on Proof by Exhaustion

More on Proof by Exhaustion

When to use Proof by Exhaustion?

More on Proof by Exhaustion

When to use Proof by Exhaustion?

When extra information in each case helps move the proof forward.

More on Proof by Exhaustion

When to use Proof by Exhaustion?

When extra information in each case helps move the proof forward.

Do we always need to prove all the cases?

More on Proof by Exhaustion

When to use Proof by Exhaustion?

When extra information in each case helps move the proof forward.

Do we always need to prove all the cases?

No.

More on Proof by Exhaustion

When to use Proof by Exhaustion?

When extra information in each case helps move the proof forward.

Do we always need to prove all the cases?

No. Often proof of one case follows by making small changes to the proof of another case.

More on Proof by Exhaustion

When to use Proof by Exhaustion?

When extra information in each case helps move the proof forward.

Do we always need to prove all the cases?

No. Often proof of one case follows by making small changes to the proof of another case.

Example: On the next slide.

Leaving Cases: Proof by Exhaustion

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof:

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

So,

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

So,

$$xy$$

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

So,

$$xy = (2k + 1)(2k' + 1)$$

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

So,

$$xy = (2k + 1)(2k' + 1) = 4kk' + 2k + 2k' + 1$$

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

So,

$$xy = (2k+1)(2k'+1) = 4kk' + 2k + 2k' + 1 = 2(2kk' + k + k') + 1$$

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

So,

$$xy \;=\; (2k+1)(2k'+1) \;=\; 4kk' + 2k + 2k' + 1 \;=\; 2(2kk' + k + k') + 1$$

Thus, $xy = 2k'' + 1$, where k'' is an integer.

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

So,

$$xy = (2k + 1)(2k' + 1) = 4kk' + 2k + 2k' + 1 = 2(2kk' + k + k') + 1$$

Thus, $xy = 2k'' + 1$, where k'' is an integer. Hence, xy is an odd integer.

Leaving Cases: Proof by Exhaustion

Theorem: Suppose $x, y \in \mathbb{Z}$. If both xy and $x + y$ are even, then both x and y are even.

Proof: We will prove the contrapositive of the theorem. That is,

Suppose $x, y \in \mathbb{Z}$. If x or y are odd, then xy or $x + y$ are odd.

Case 1: Assume both x and y are odd.

By the definition of an odd integer, $x = 2k + 1$ and $y = 2k' + 1$, for $k, k' \in \mathbb{Z}$.

So,

$$xy = (2k + 1)(2k' + 1) = 4kk' + 2k + 2k' + 1 = 2(2kk' + k + k') + 1$$

Thus, $xy = 2k'' + 1$, where k'' is an integer. Hence, xy is an odd integer.

…

Leaving Cases: Proof by Exhaustion

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

$$x + y$$

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality* (*WLOG*) assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

$$x + y \;=\; 2k + 1 + 2k'$$

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

$$x + y \;=\; 2k + 1 + 2k' \;=\; 2(k + k') + 1$$

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

$$x + y \;=\; 2k + 1 + 2k' \;=\; 2(k + k') + 1$$

Thus, $x + y = 2k'' + 1$, where k'' is an integer.

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality* (*WLOG*) assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

$$x + y \;=\; 2k + 1 + 2k' \;=\; 2(k + k') + 1$$

Thus, $x + y = 2k'' + 1$, where k'' is an integer. Hence, $x + y$ is an odd integer.

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

$$x + y \;=\; 2k + 1 + 2k' \;=\; 2(k + k') + 1$$

Thus, $x + y = 2k'' + 1$, where k'' is an integer. Hence, $x + y$ is an odd integer. ∎

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

$$x + y \;=\; 2k + 1 + 2k' \;=\; 2(k + k') + 1$$

Thus, $x + y = 2k'' + 1$, where k'' is an integer. Hence, $x + y$ is an odd integer. $\blacksquare$

Note: A third case, where x is even and y is odd, is not required because proof for that case

Leaving Cases: Proof by Exhaustion

Case 2: *Without loss of generality (WLOG)* assume that x is odd and y is even.

By the definition of an odd integer, $x = 2k + 1$, for some integer k.

By the definition of an even integer, $y = 2k'$, for some integer k'.

So,

$$x + y \;=\; 2k + 1 + 2k' \;=\; 2(k + k') + 1$$

Thus, $x + y = 2k'' + 1$, where k'' is an integer. Hence, $x + y$ is an odd integer. ■

Note: A third case, where x is even and y is odd, is not required because proof for that case is the same as the proof of Case 2 where x and y are interchanged.

Existence Proofs

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$ that actually gives a c, such that $P(c)$ is true

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$ that actually gives a c, such that $P(c)$ is true is called constructive proof.

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$ that actually gives a c, such that $P(c)$ is true is called constructive proof.

Example: For a non-zero rational number r, we actually constructed two irrational

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$ that actually gives a c, such that $P(c)$ is true is called constructive proof.

Example: For a non-zero rational number r, we actually constructed two irrational numbers, $\sqrt{2}$ and $\frac{r}{\sqrt{2}}$, whose product is r.

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$ that actually gives a c, such that $P(c)$ is true is called constructive proof.

Example: For a non-zero rational number r, we actually constructed two irrational numbers, $\sqrt{2}$ and $\frac{r}{\sqrt{2}}$, whose product is r.

Defn: An existence proof of proposition $\exists x P(x)$

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$ that actually gives a c, such that $P(c)$ is true is called constructive proof.

Example: For a non-zero rational number r, we actually constructed two irrational numbers, $\sqrt{2}$ and $\frac{r}{\sqrt{2}}$, whose product is r.

Defn: An existence proof of proposition $\exists x P(x)$ that proves the existence of a c for which $P(c)$ is

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$ that actually gives a c, such that $P(c)$ is true is called constructive proof.

Example: For a non-zero rational number r, we actually constructed two irrational numbers, $\sqrt{2}$ and $\dfrac{r}{\sqrt{2}}$, whose product is r.

Defn: An existence proof of proposition $\exists x P(x)$ that proves the existence of a c for which $P(c)$ is true without actually giving a c is called nonconstructive proof.

Existence Proofs

Defn: A proof of a proposition such as $\exists x P(x)$ (or $\exists x \exists y P(x, y)$, etc.) is called an existence proof.

Defn: An existence proof of proposition $\exists x P(x)$ that actually gives a c, such that $P(c)$ is true is called constructive proof.

Example: For a non-zero rational number r, we actually constructed two irrational numbers, $\sqrt{2}$ and $\frac{r}{\sqrt{2}}$, whose product is r.

Defn: An existence proof of proposition $\exists x P(x)$ that proves the existence of a c for which $P(c)$ is true without actually giving a c is called nonconstructive proof.

Example: On the next slide.

Example: Non-constructive Existence Proof

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof:

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational.

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational,

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = y = \sqrt{2}$.

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is irrational,

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is irrational, then we can let $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ because,

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is irrational, then we can let $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ because,

$$x^y$$

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is irrational, then we can let $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ because,

$$x^y = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$$

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is irrational, then we can let $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ because,

$$x^y = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^2$$

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is irrational, then we can let $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ because,

$$x^y = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^2 = 2$$

Example: Non-constructive Existence Proof

Theorem: There exist irrational numbers x and y such that x^y is rational.

Proof: We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$.

If $\sqrt{2}^{\sqrt{2}}$ is rational, then $x = y = \sqrt{2}$.

If $\sqrt{2}^{\sqrt{2}}$ is irrational, then we can let $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$ because,

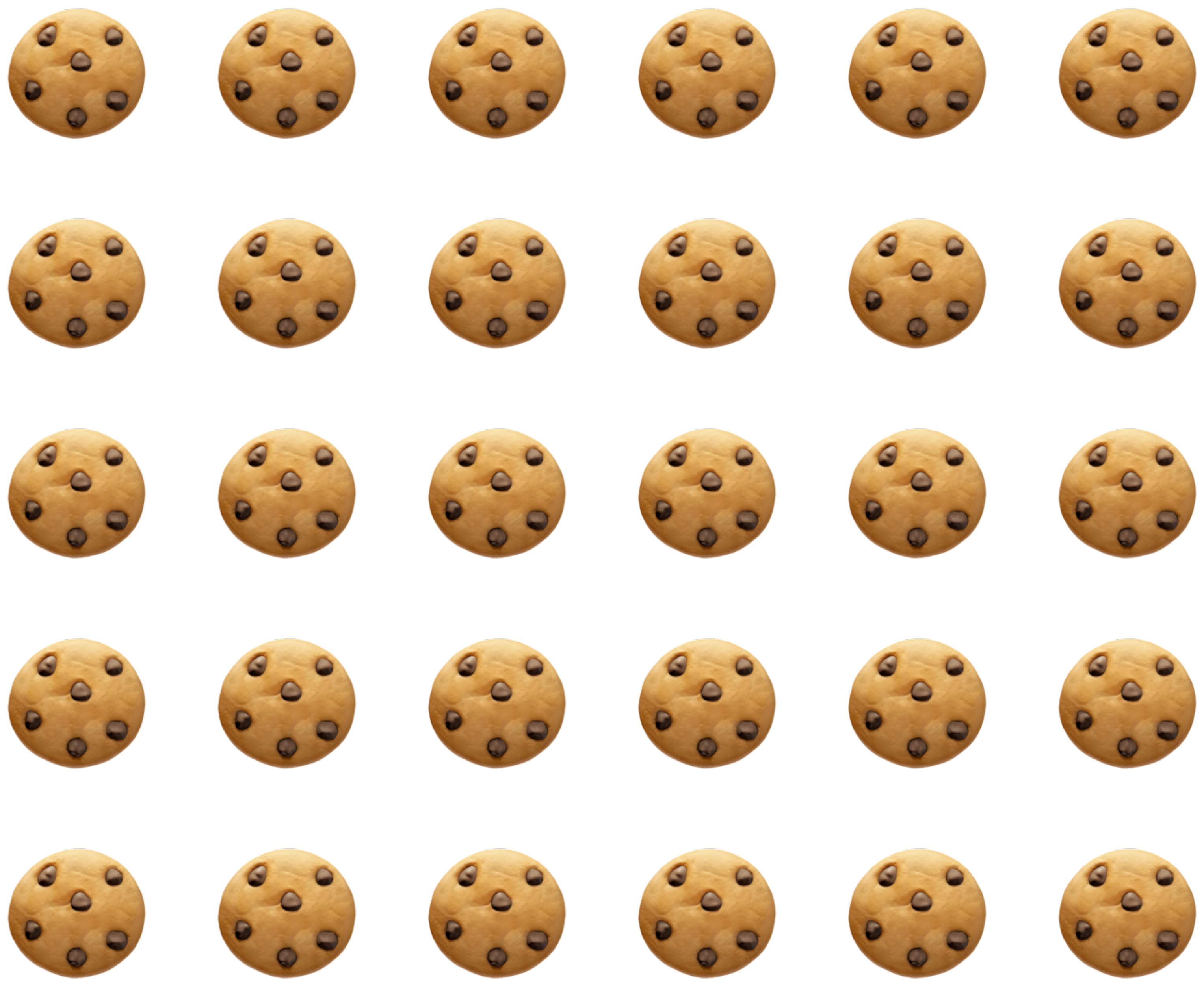
$$x^y = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^2 = 2$$

∎

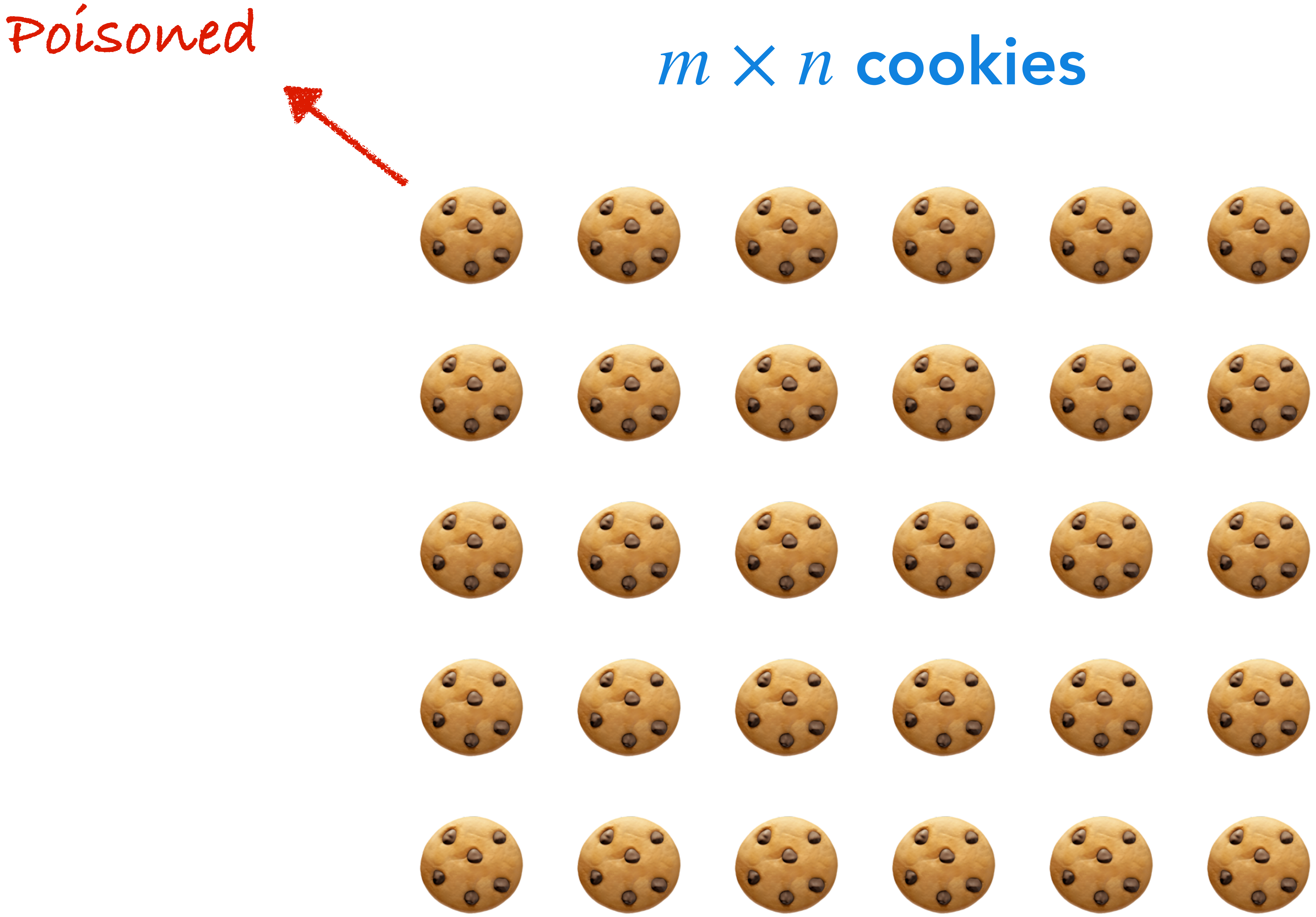
Game of Chomp

Game of Chomp

$m \times n$ cookies

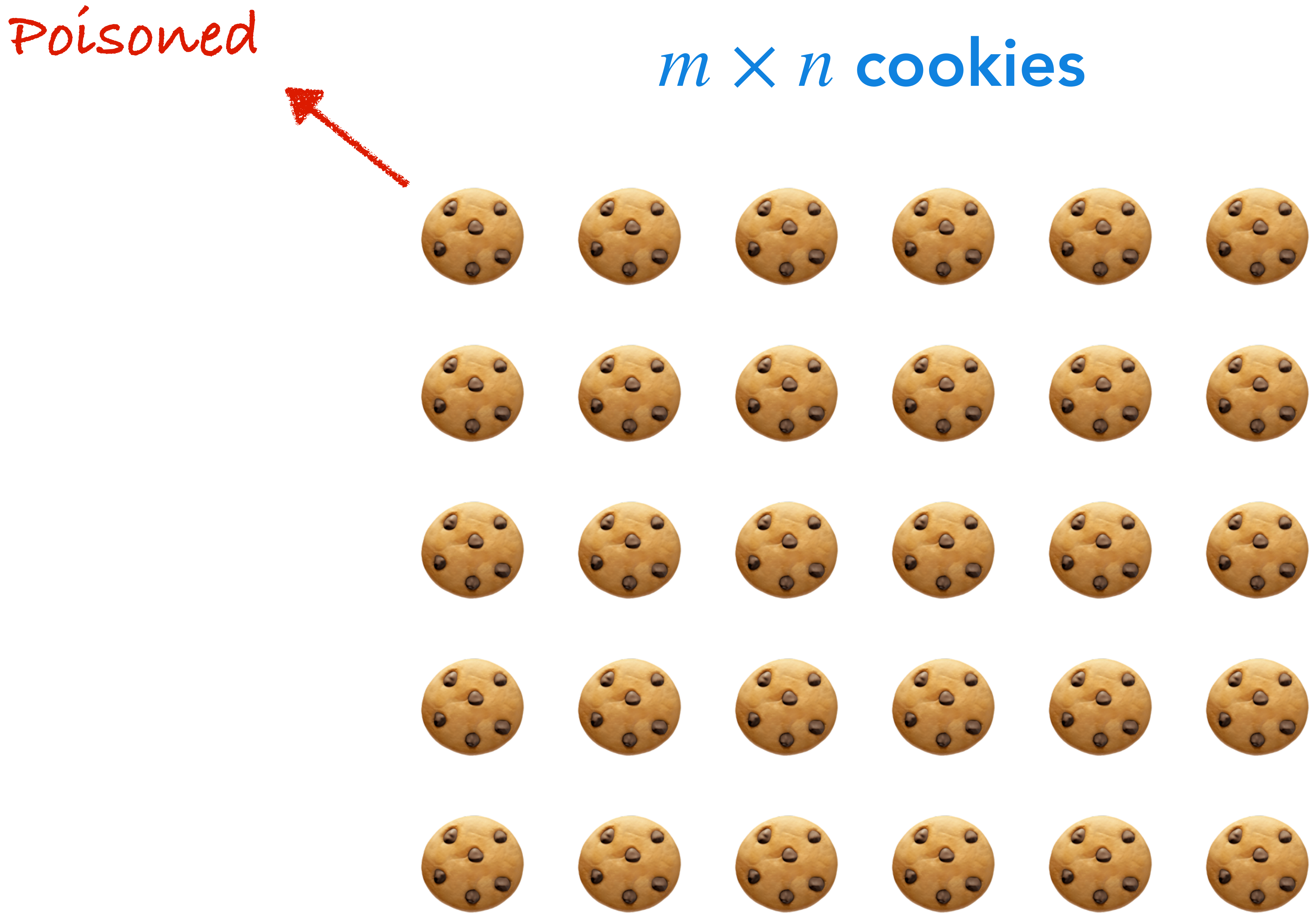


Game of Chomp



Game of Chomp

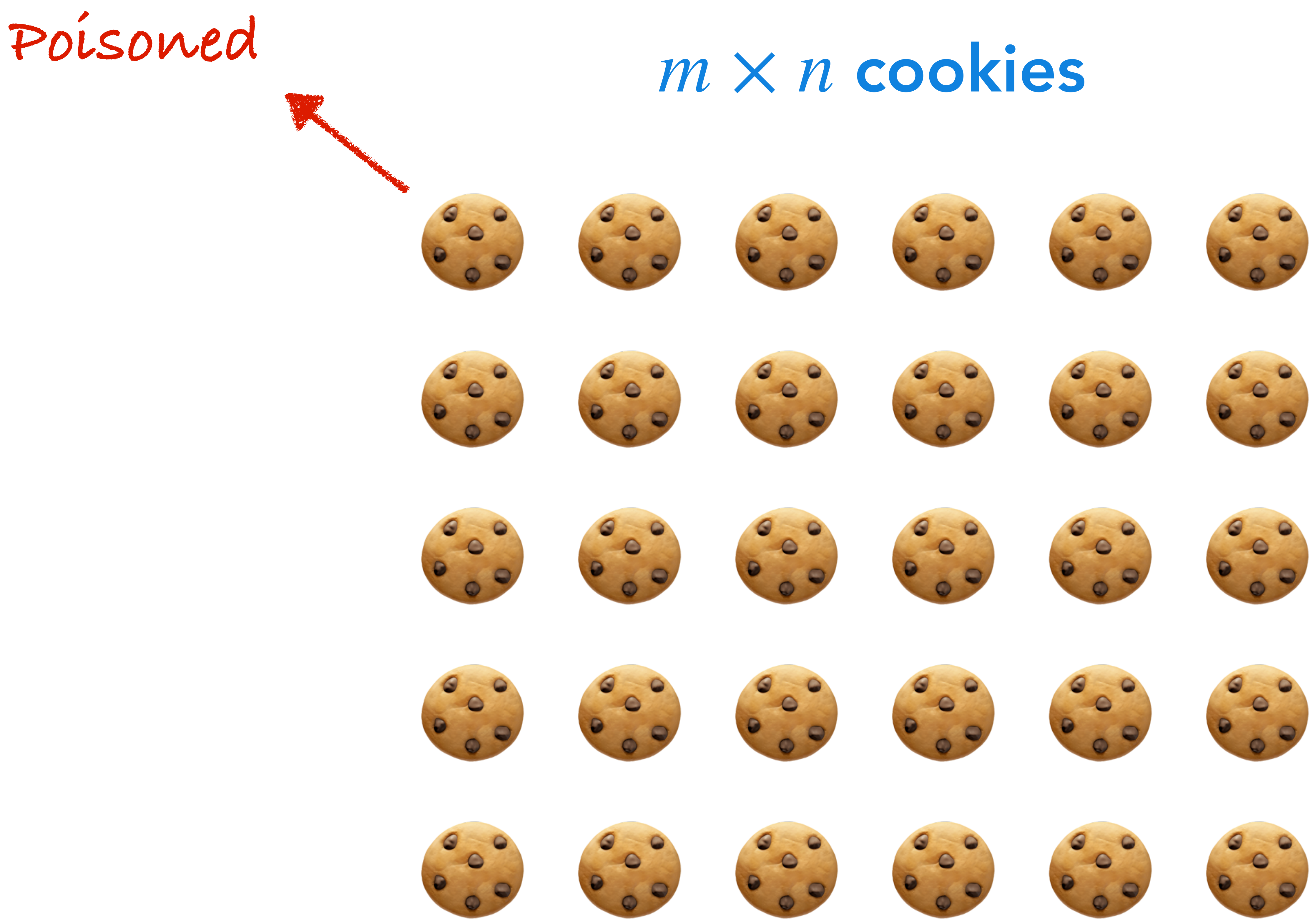
Rules:



Game of Chomp

Rules:

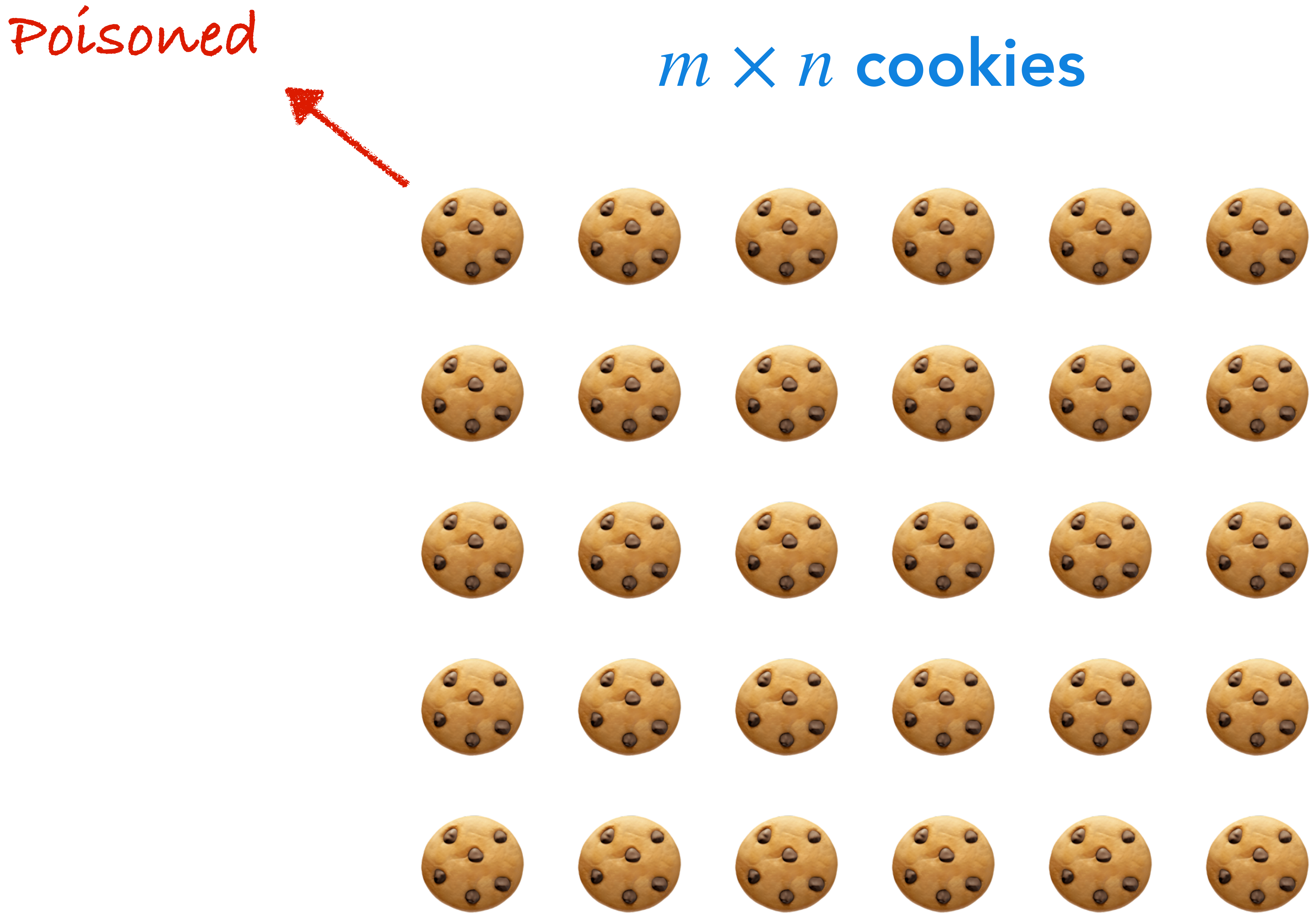
- 2-player game.



Game of Chomp

Rules:

- 2-player game.
- A player on her turn can pick a cookie.

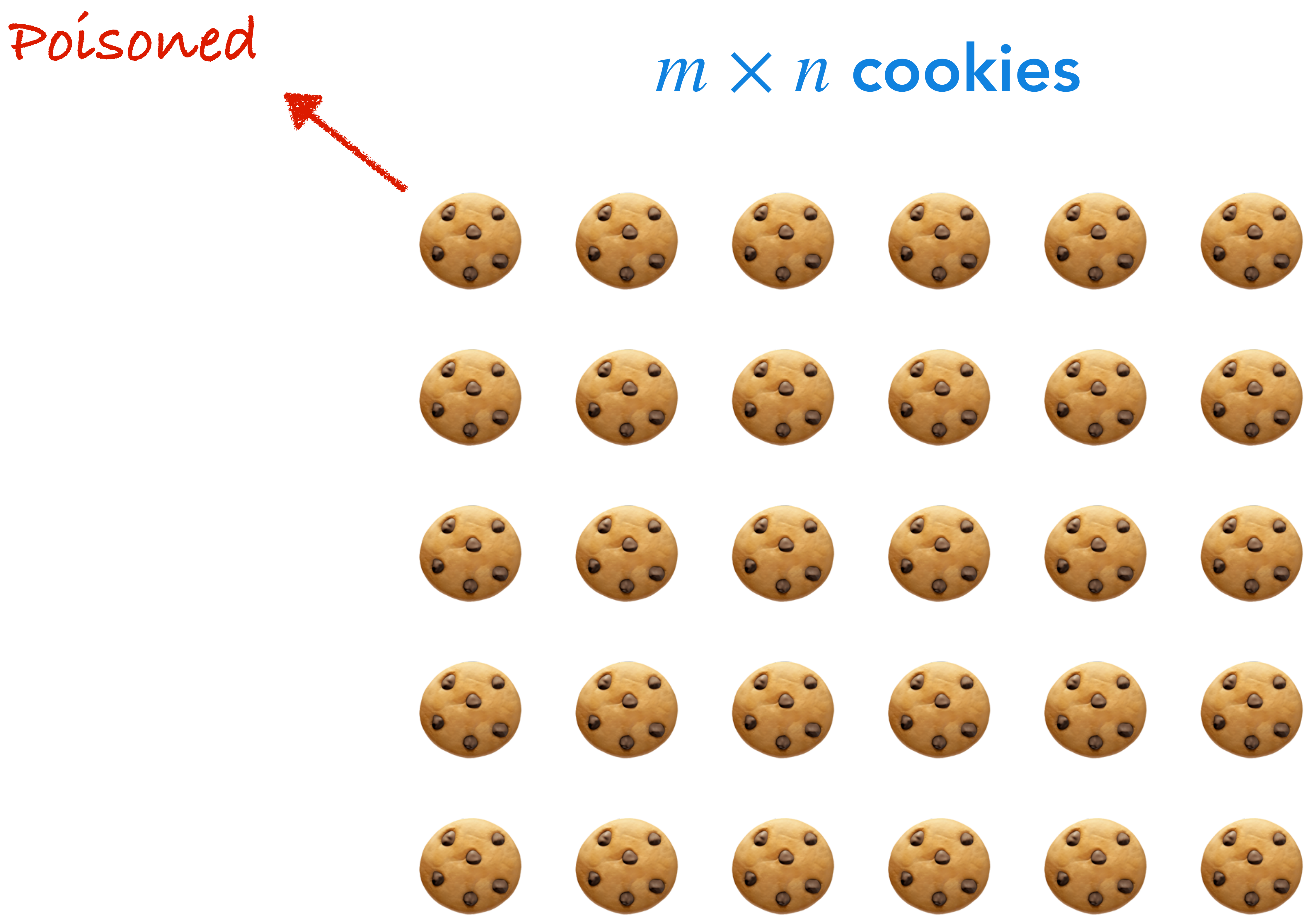


Game of Chomp

Rules:

- 2-player game.
- A player on her turn can pick a cookie.

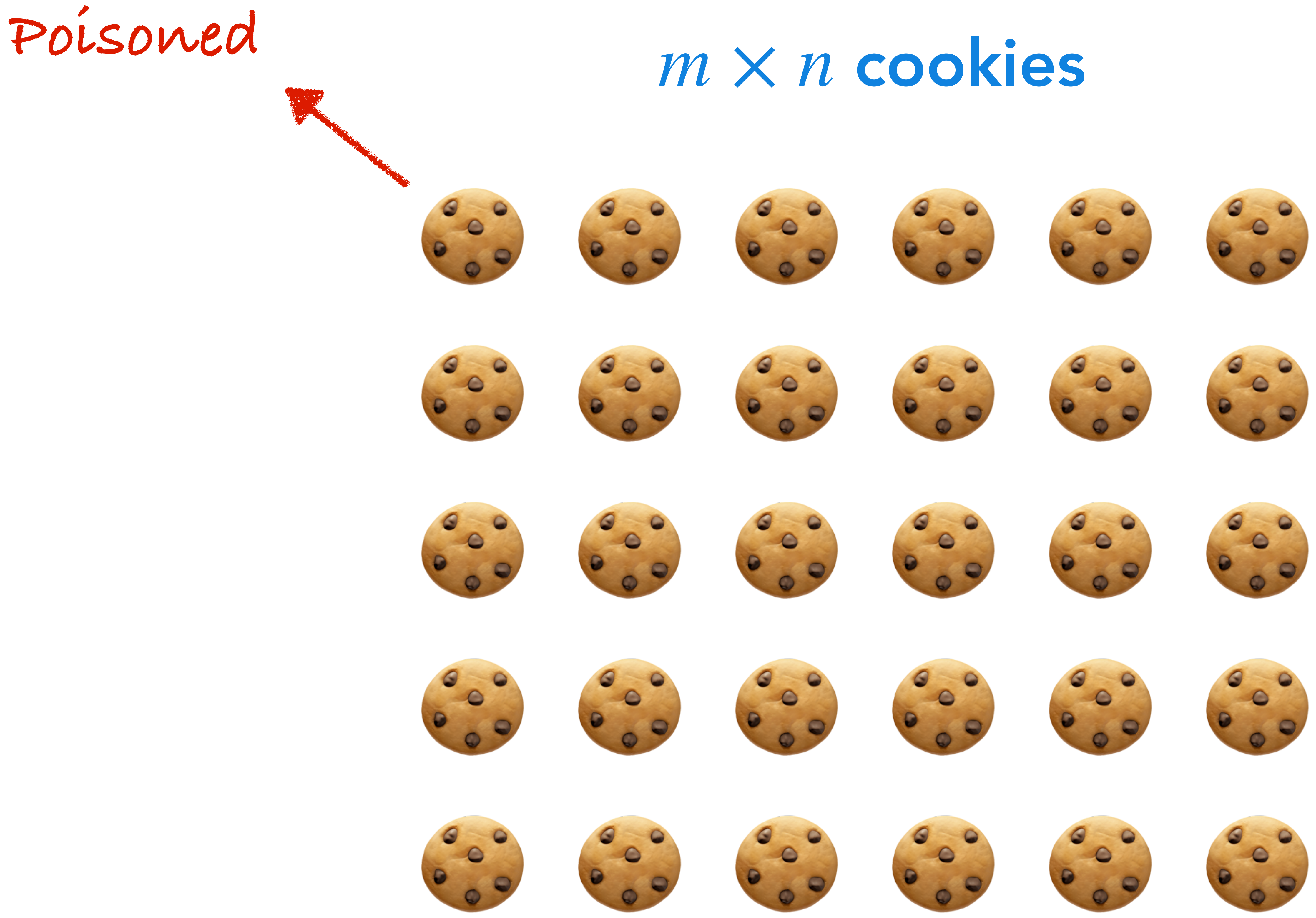
  She has to eat that cookie + cookies to



Game of Chomp

Rules:

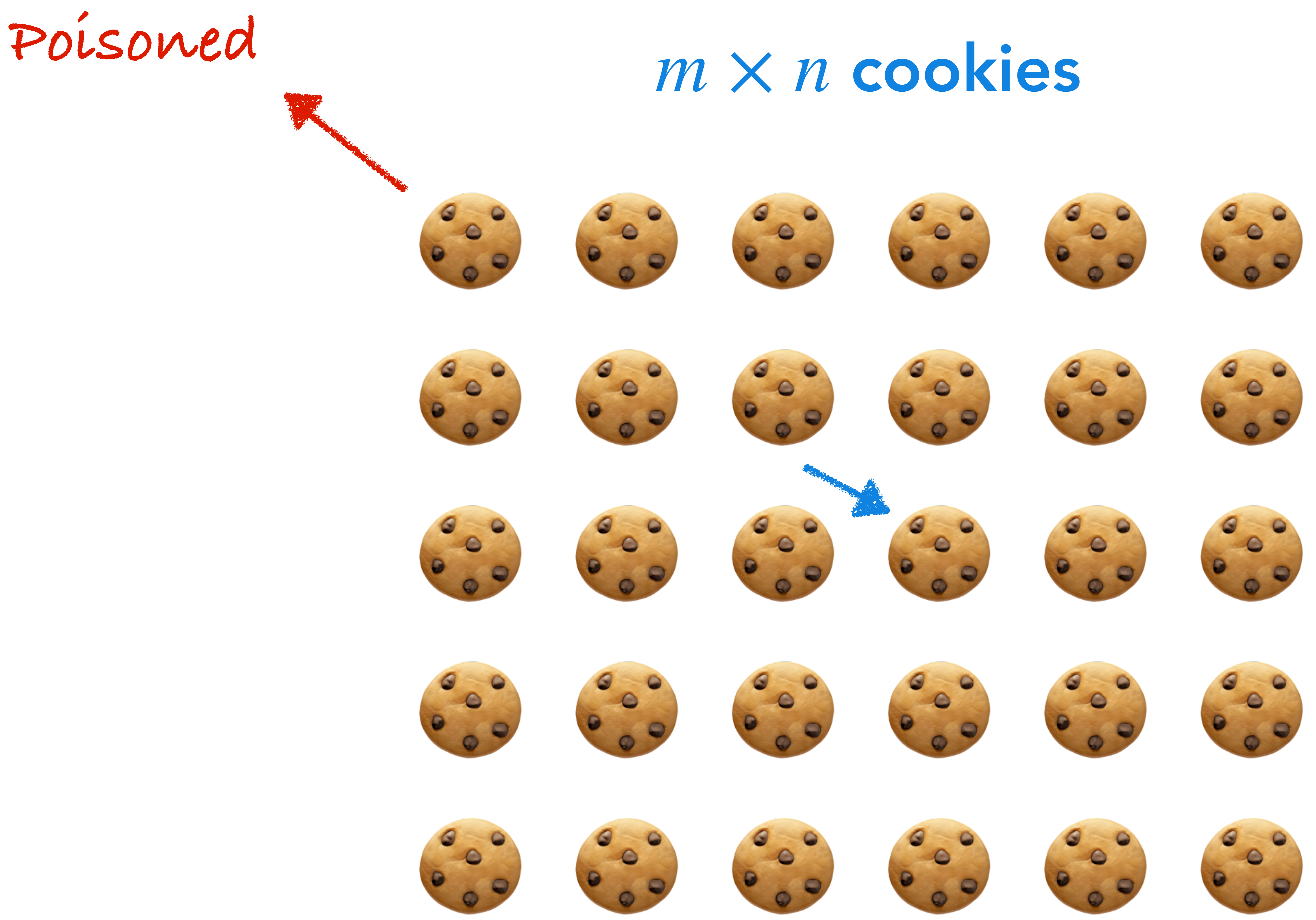
- 2-player game.
- A player on her turn can pick a cookie. She has to eat that cookie + cookies to the down and right of selected cookie.



Game of Chomp

Rules:

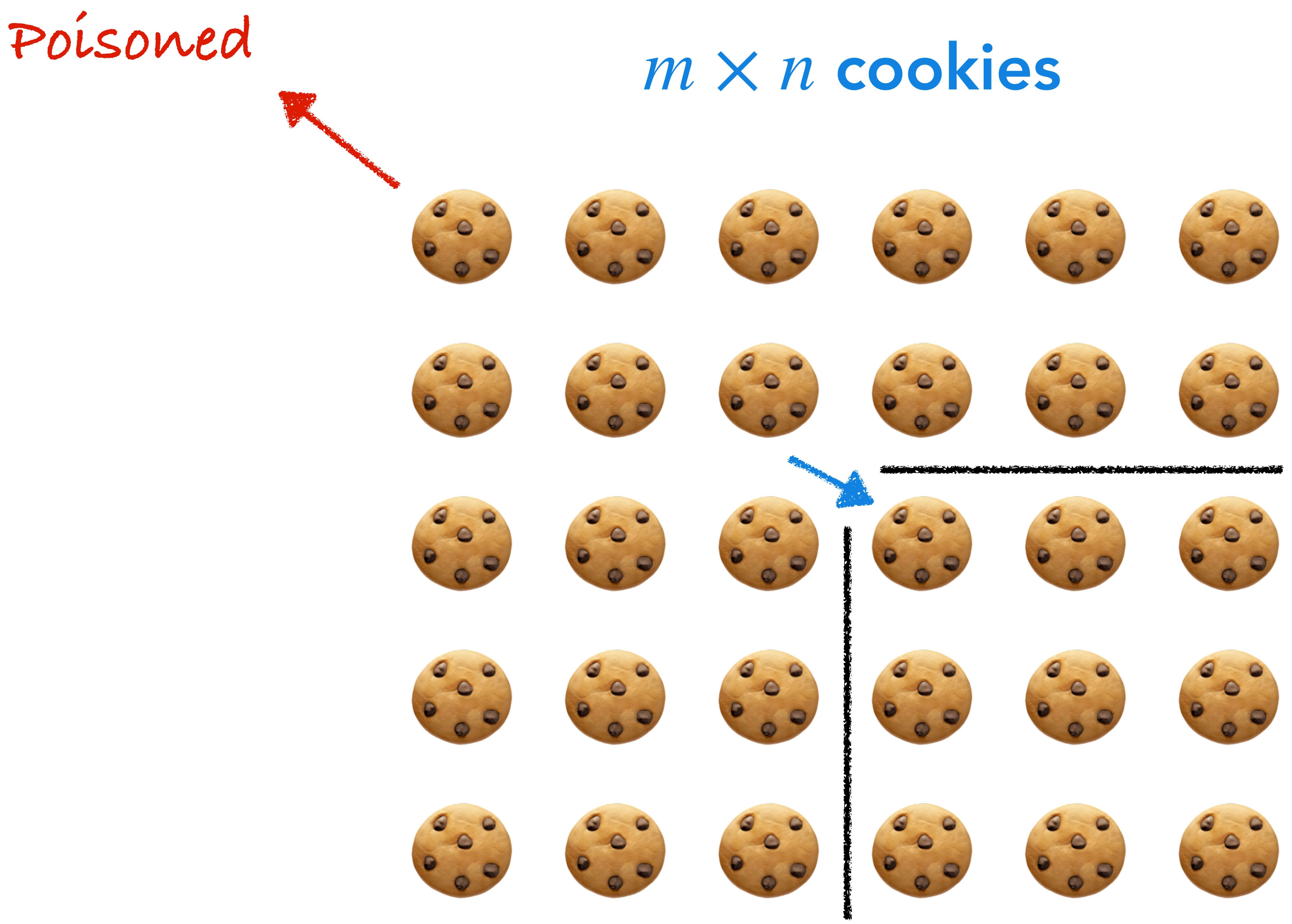
- 2-player game.
- A player on her turn can pick a cookie. She has to eat that cookie + cookies to the down and right of selected cookie.



Game of Chomp

Rules:

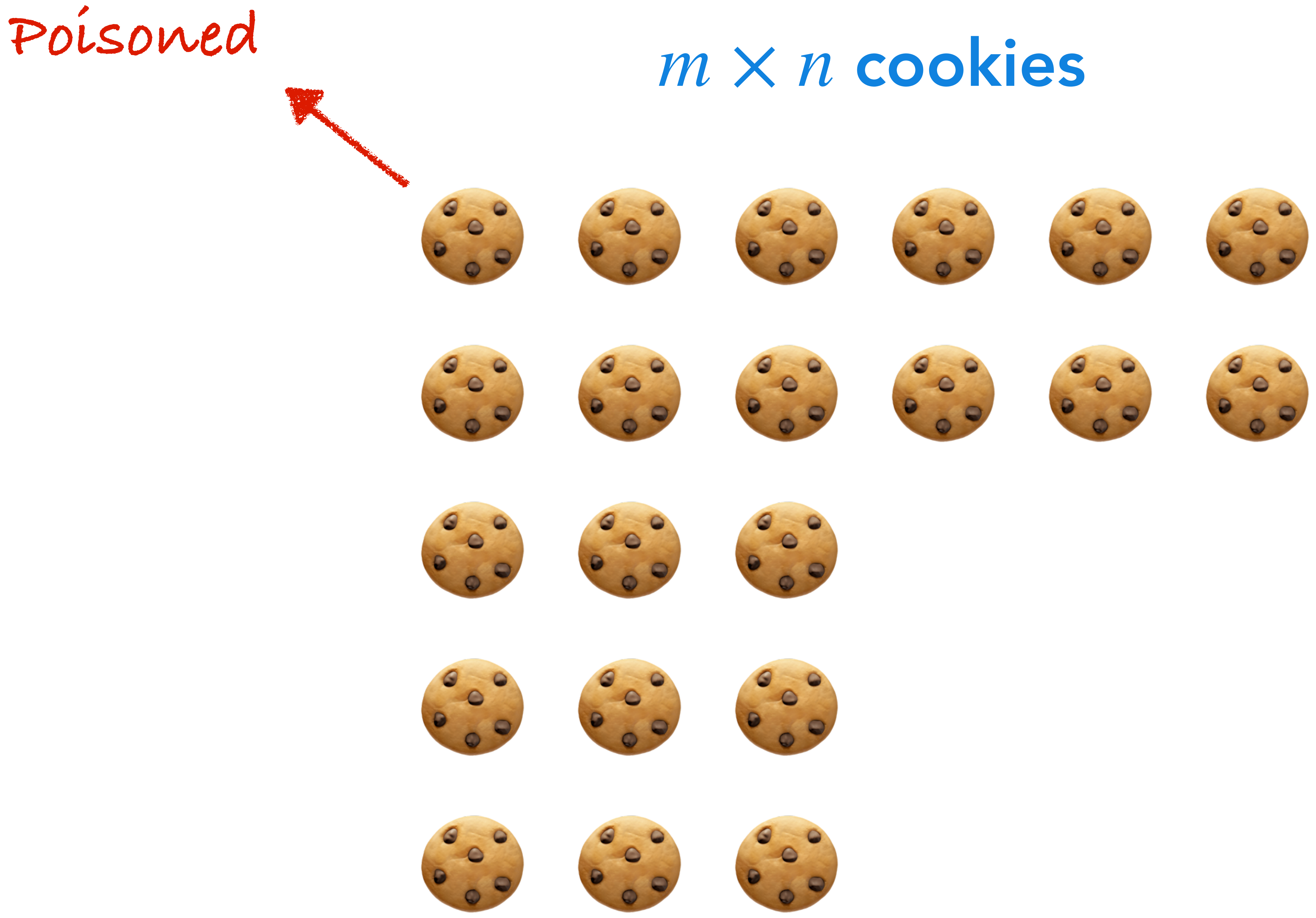
- 2-player game.
- A player on her turn can pick a cookie. She has to eat that cookie + cookies to the down and right of selected cookie.



Game of Chomp

Rules:

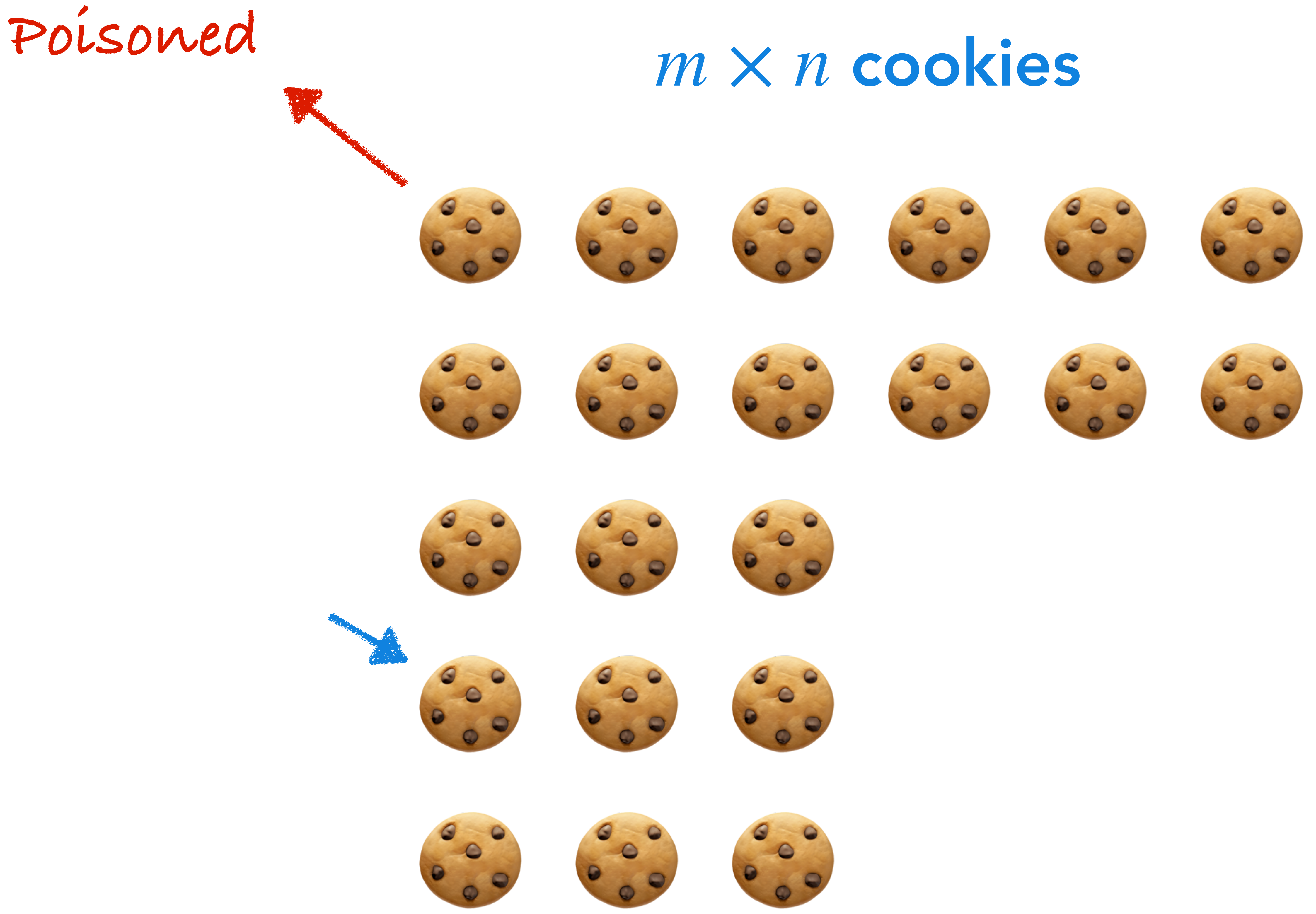
- 2-player game.
- A player on her turn can pick a cookie. She has to eat that cookie + cookies to the down and right of selected cookie.



Game of Chomp

Rules:

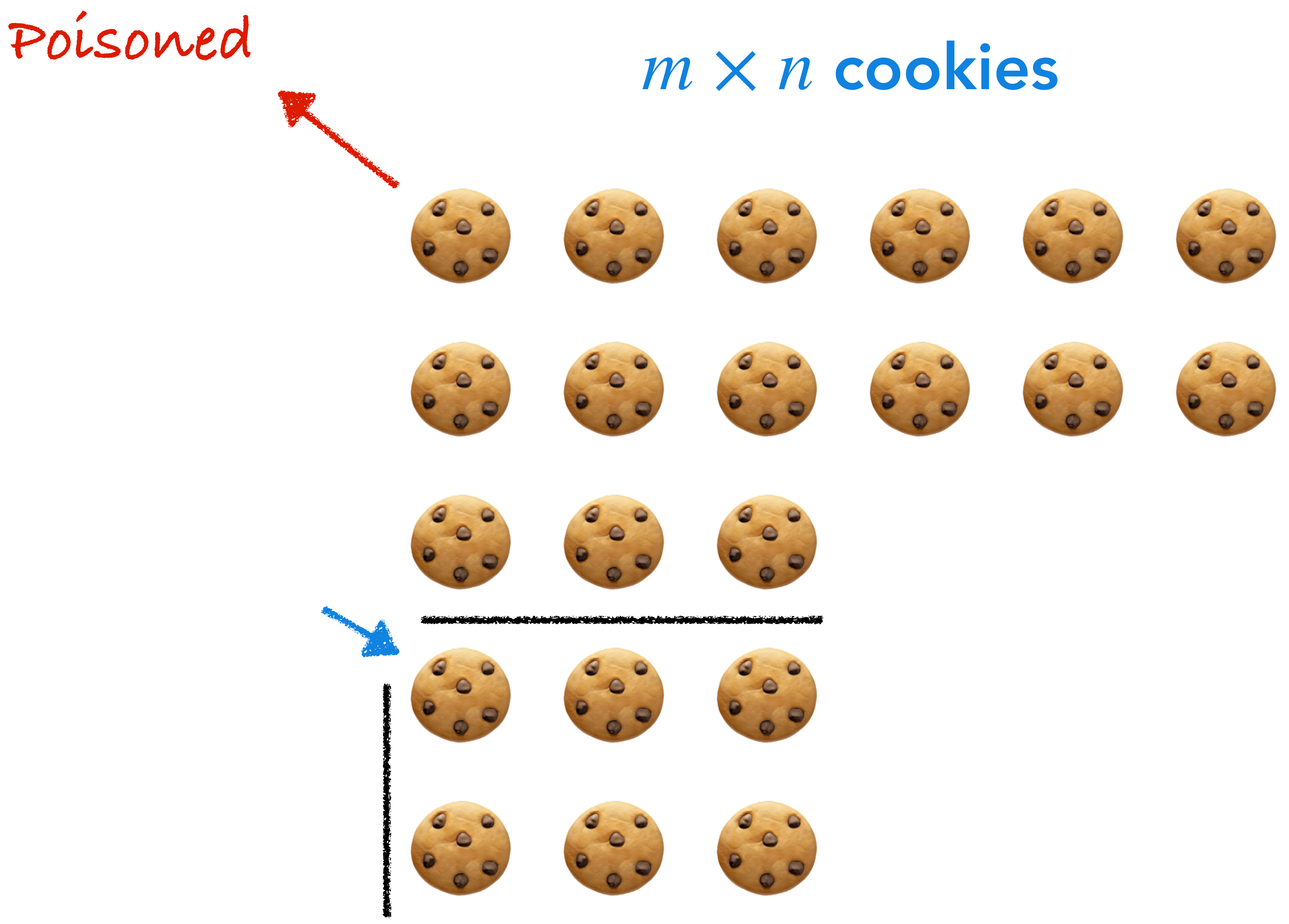
- 2-player game.
- A player on her turn can pick a cookie. She has to eat that cookie + cookies to the down and right of selected cookie.



Game of Chomp

Rules:

- 2-player game.
- A player on her turn can pick a cookie. She has to eat that cookie + cookies to the down and right of selected cookie.



Game of Chomp

Poisoned

$m \times n$ **cookies**

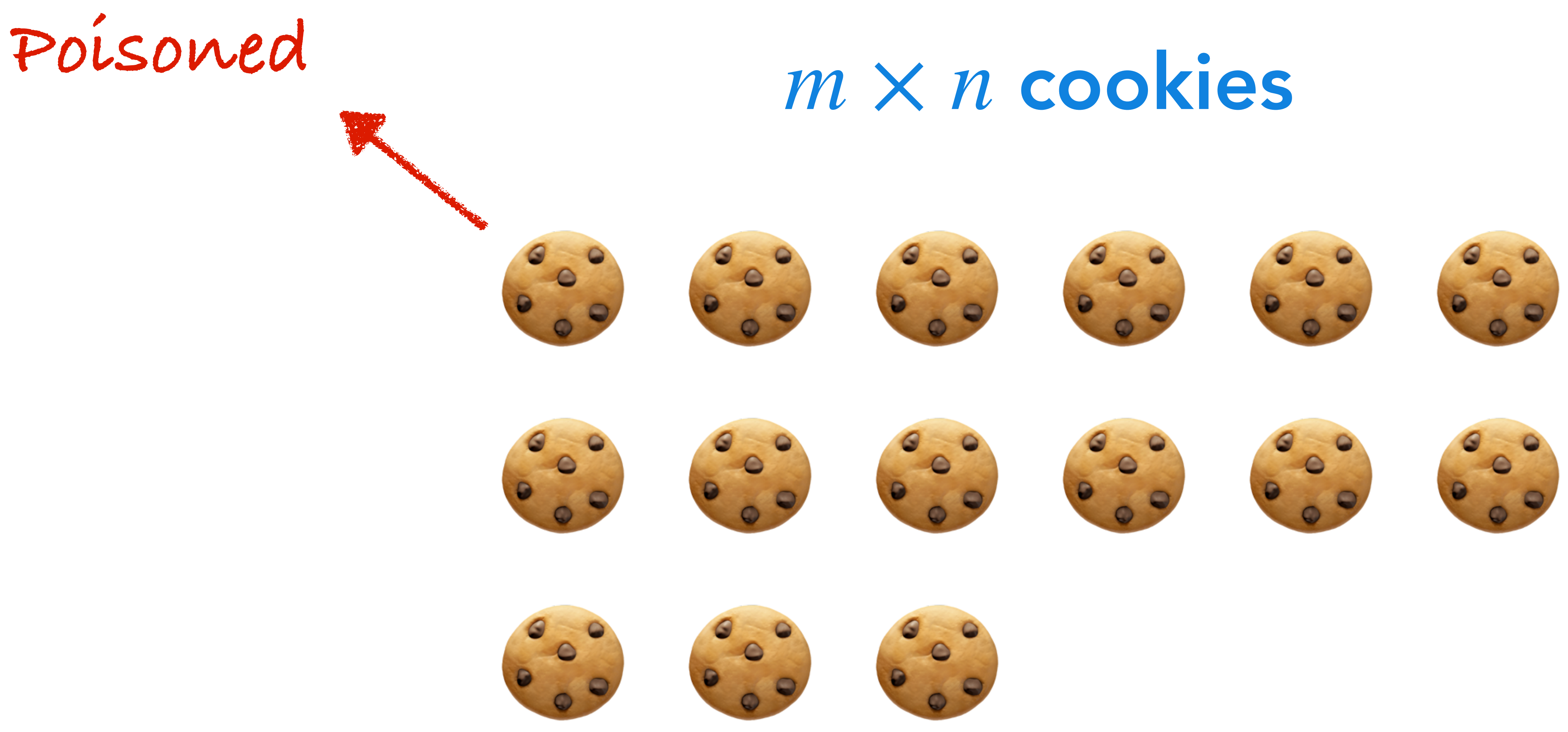
Rules:

- 2-player game.
- A player on her turn can pick a cookie. She has to eat that cookie + cookies to the down and right of selected cookie.

Game of Chomp

Rules:

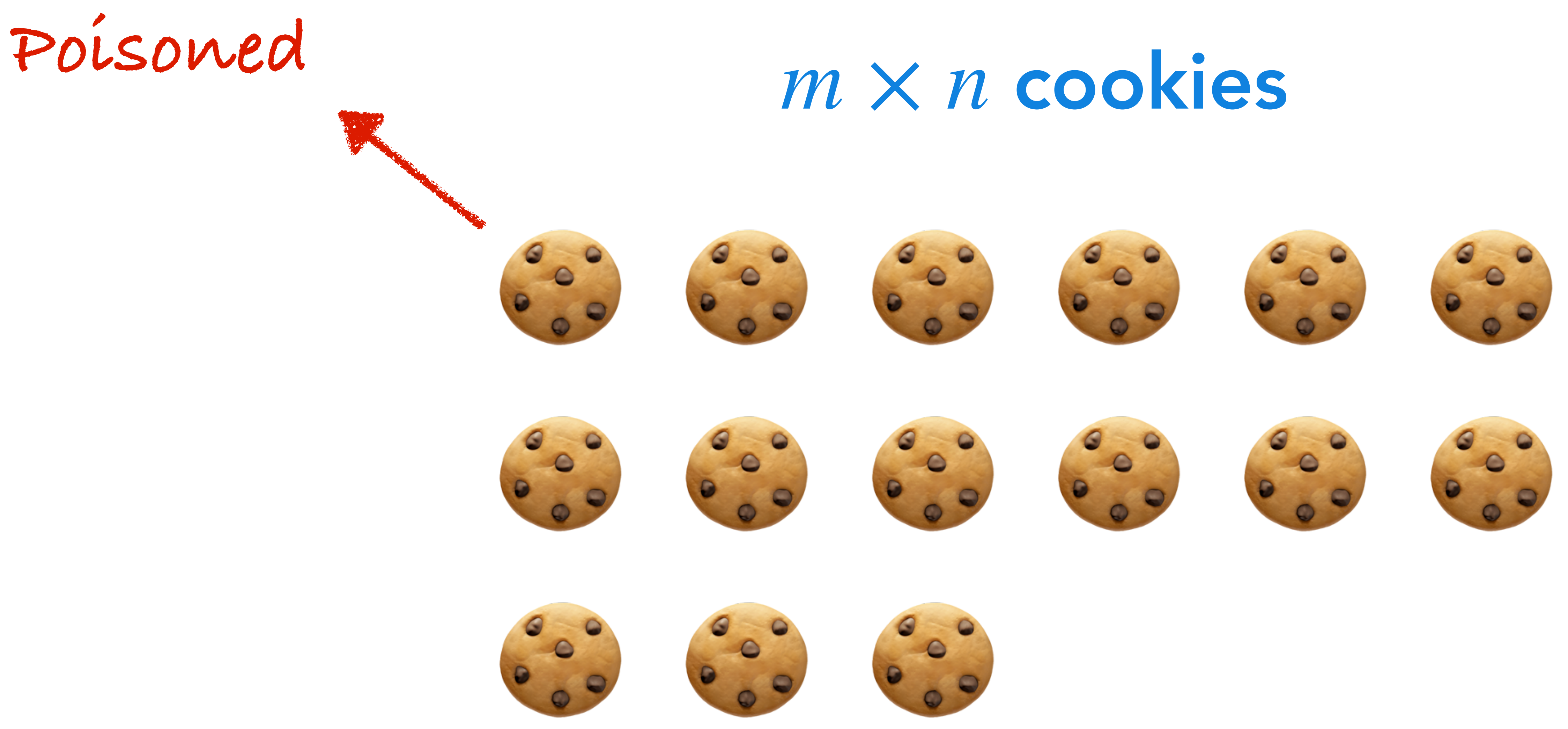
- 2-player game.
- A player on her turn can pick a cookie.
  She has to eat that cookie + cookies to
  the down and right of selected cookie.
- Player forced to eat the poisoned cookie loses.



Game of Chomp

Rules:

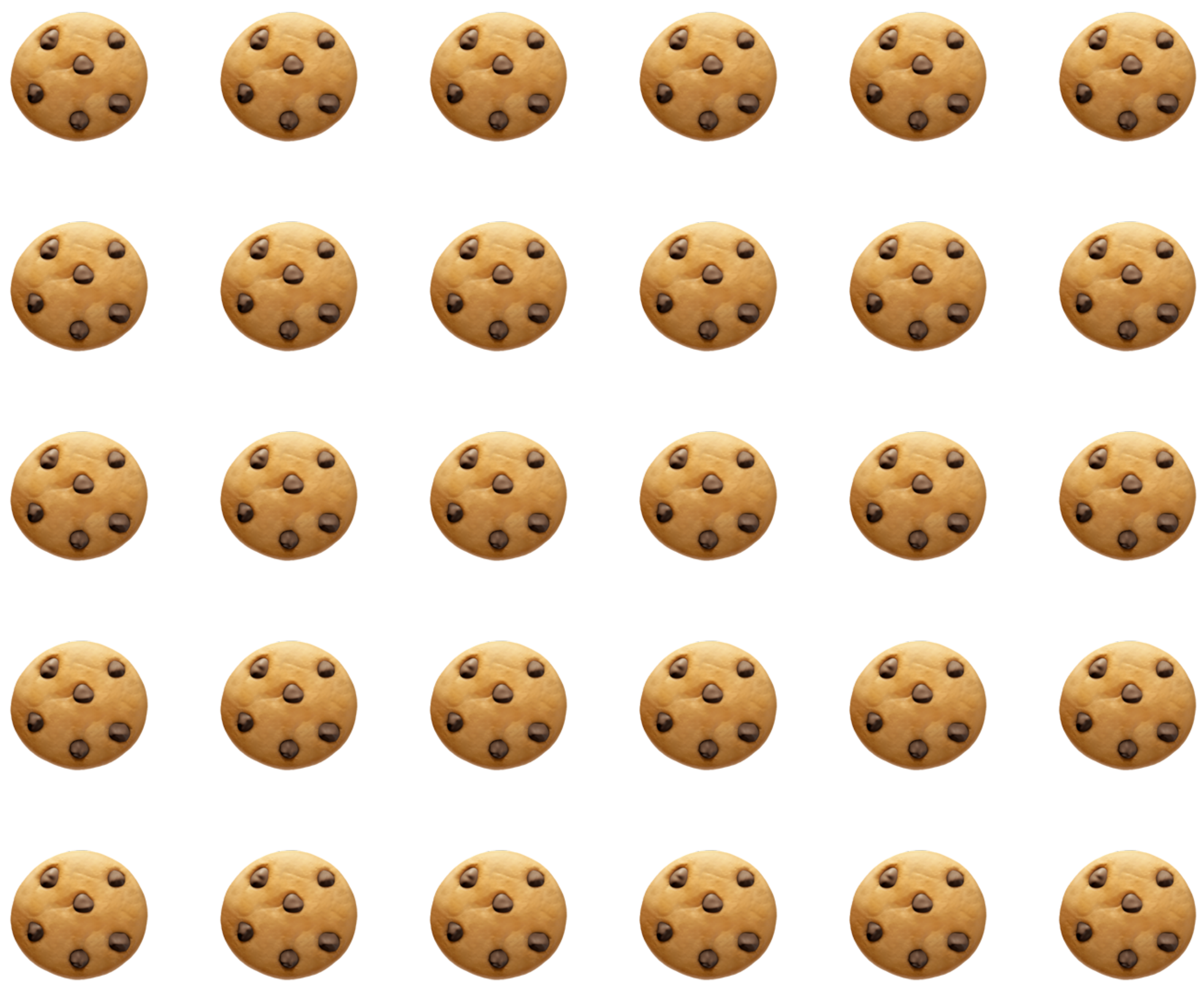
- 2-player game.
- A player on her turn can pick a cookie.
  She has to eat that cookie + cookies to
  the down and right of selected cookie.
- Player forced to eat the poisoned cookie loses.



Does the 1st player have a winning strategy?

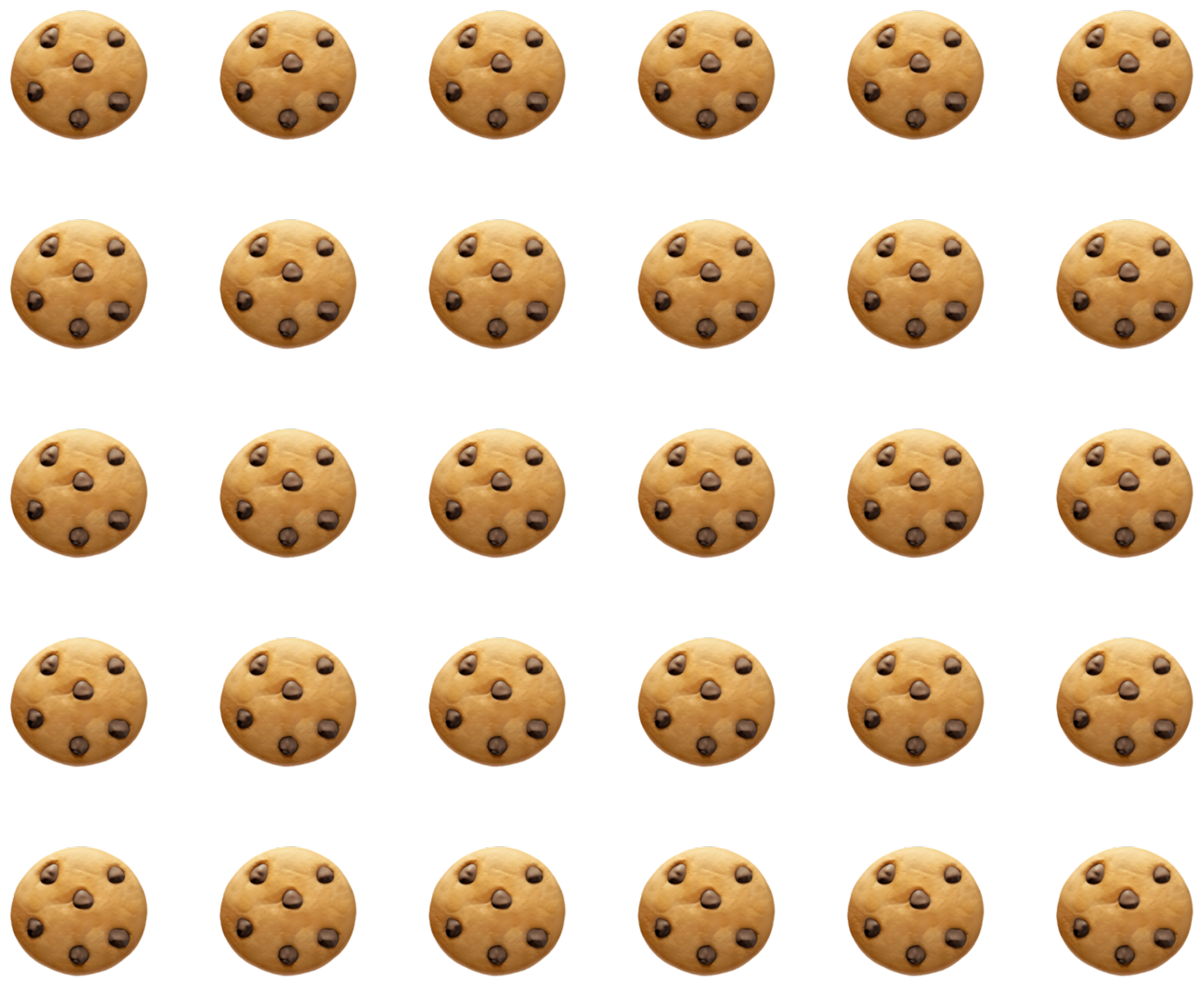
Game of Chomp

Game of Chomp



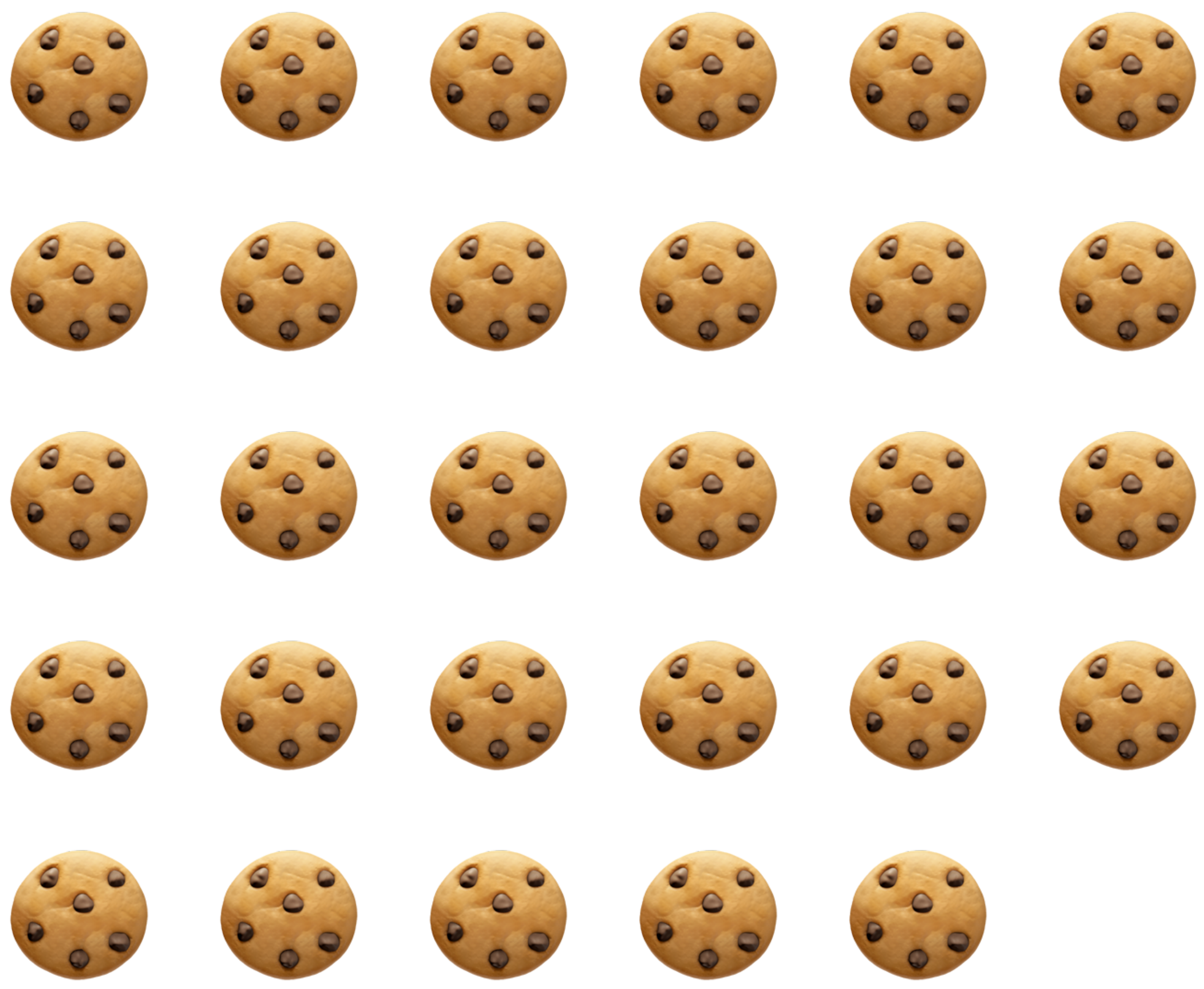
Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:



Game of Chomp

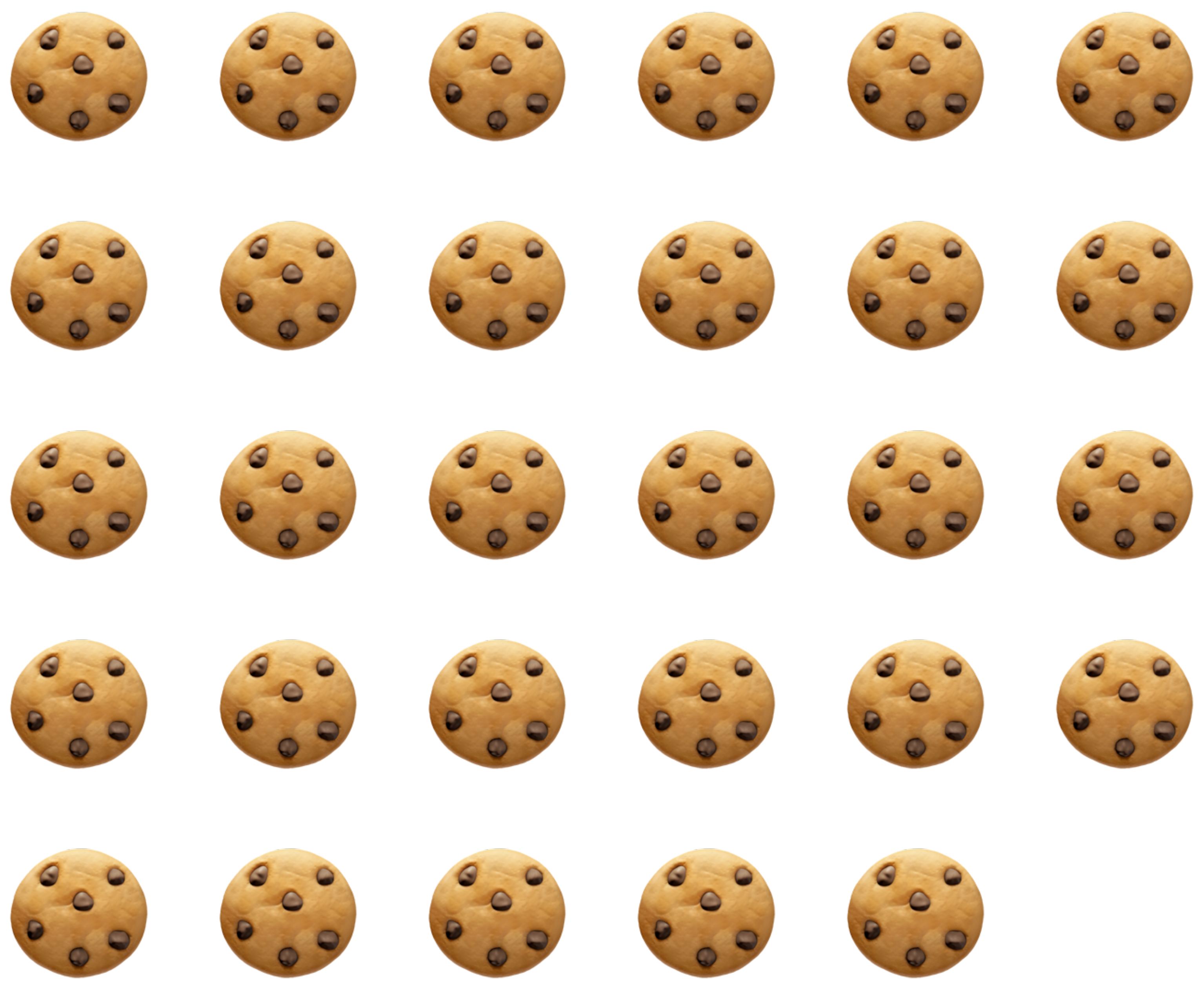
If 1st player eats the bottom-right cookie, two cases are possible:



Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

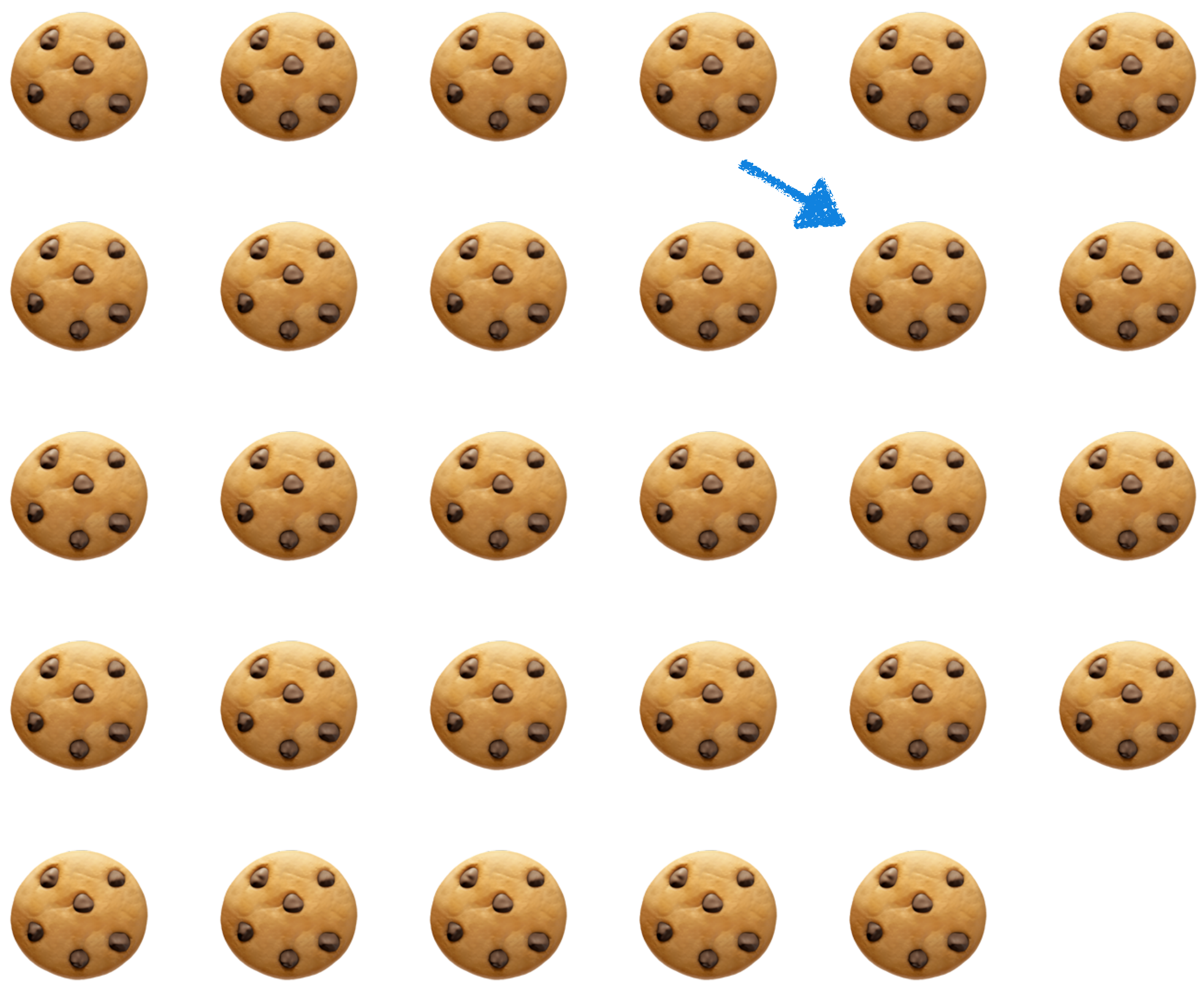
- 2nd player cannot win no matter what she tries.



Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

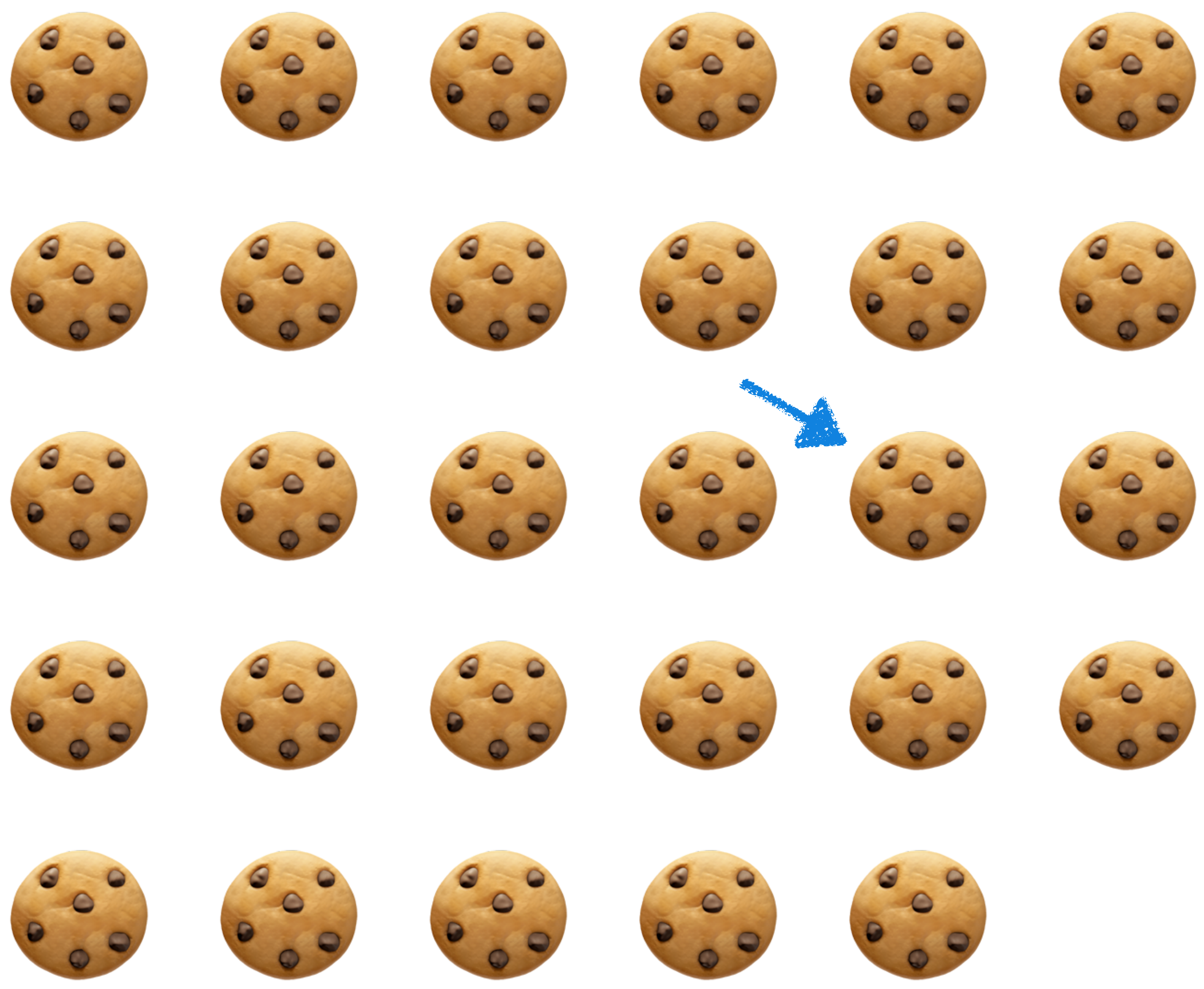
- 2nd player cannot win no matter what she tries.



Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

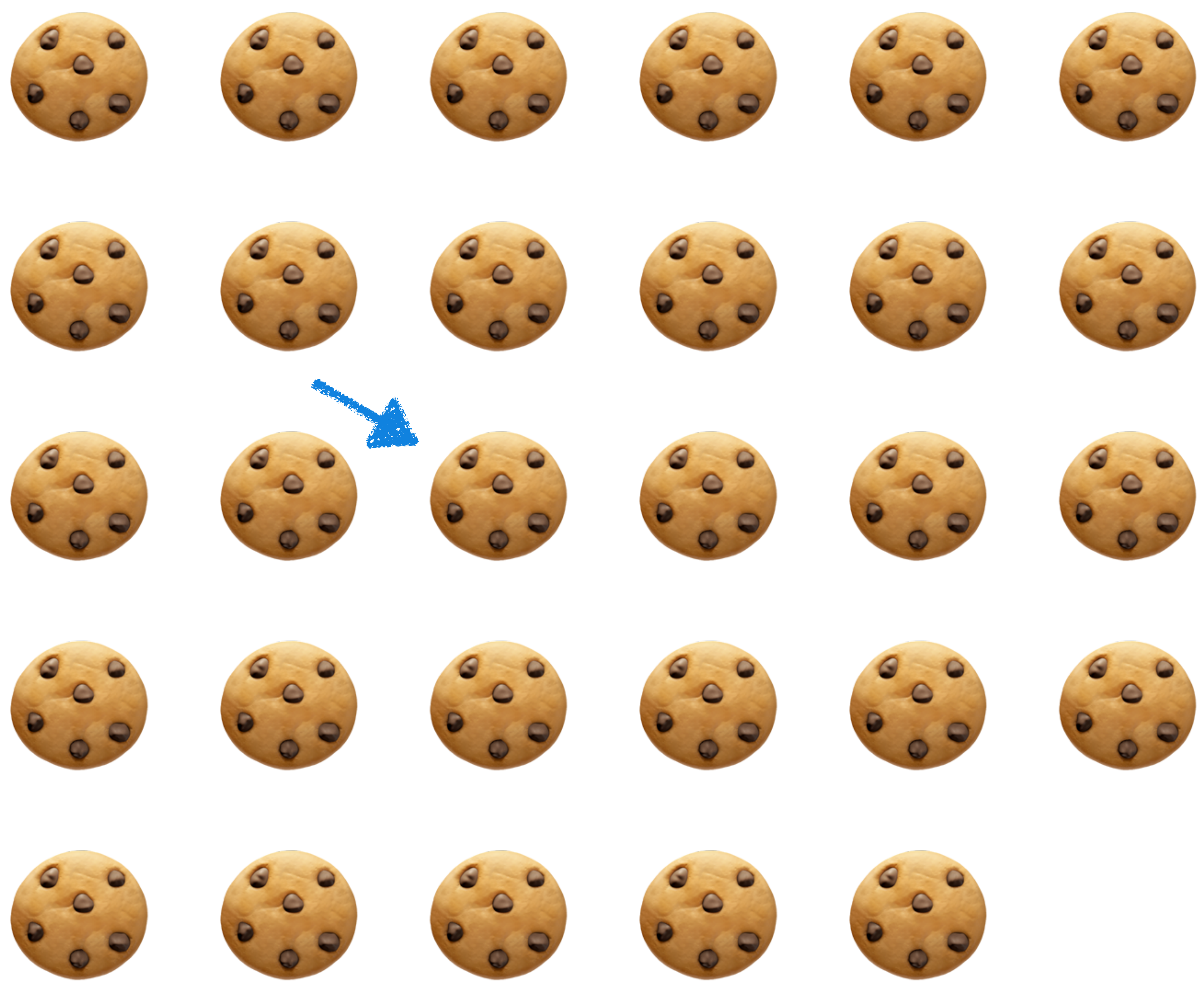
- 2nd player cannot win no matter what she tries.



Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

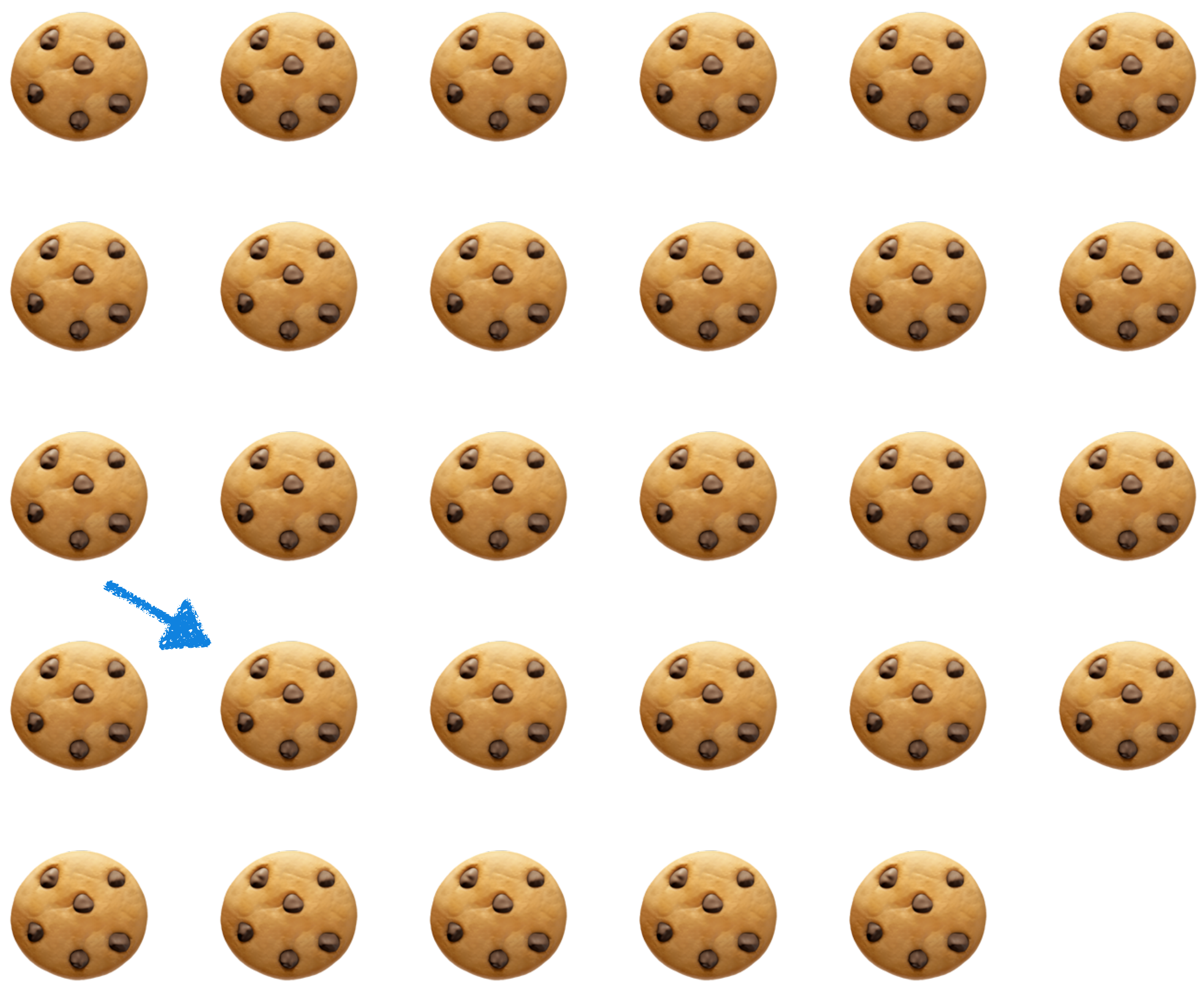
- 2nd player cannot win no matter what she tries.



Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

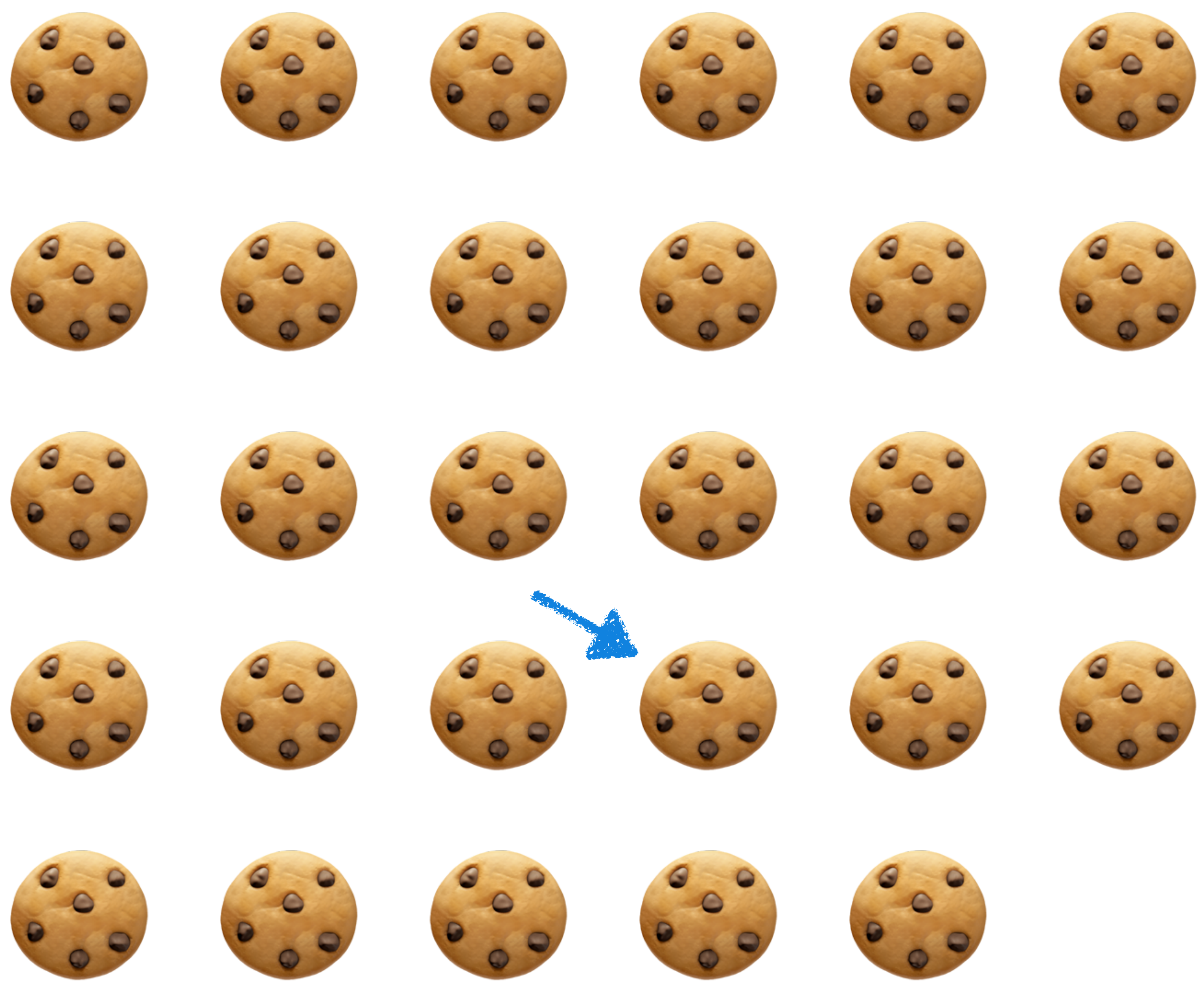
- 2nd player cannot win no matter what she tries.



Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

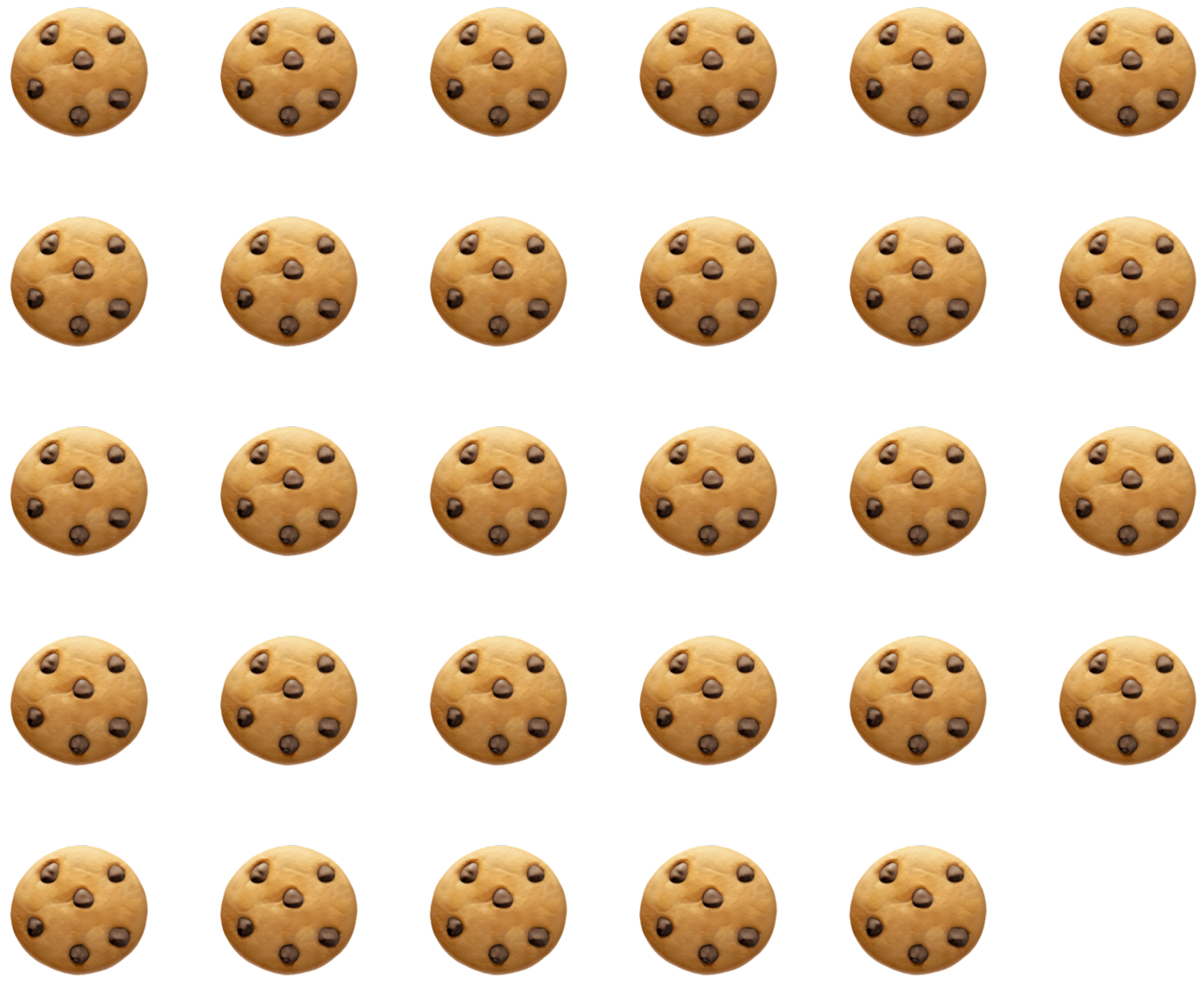
- 2nd player cannot win no matter what she tries.



Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

- 2nd player cannot win no matter what she tries.

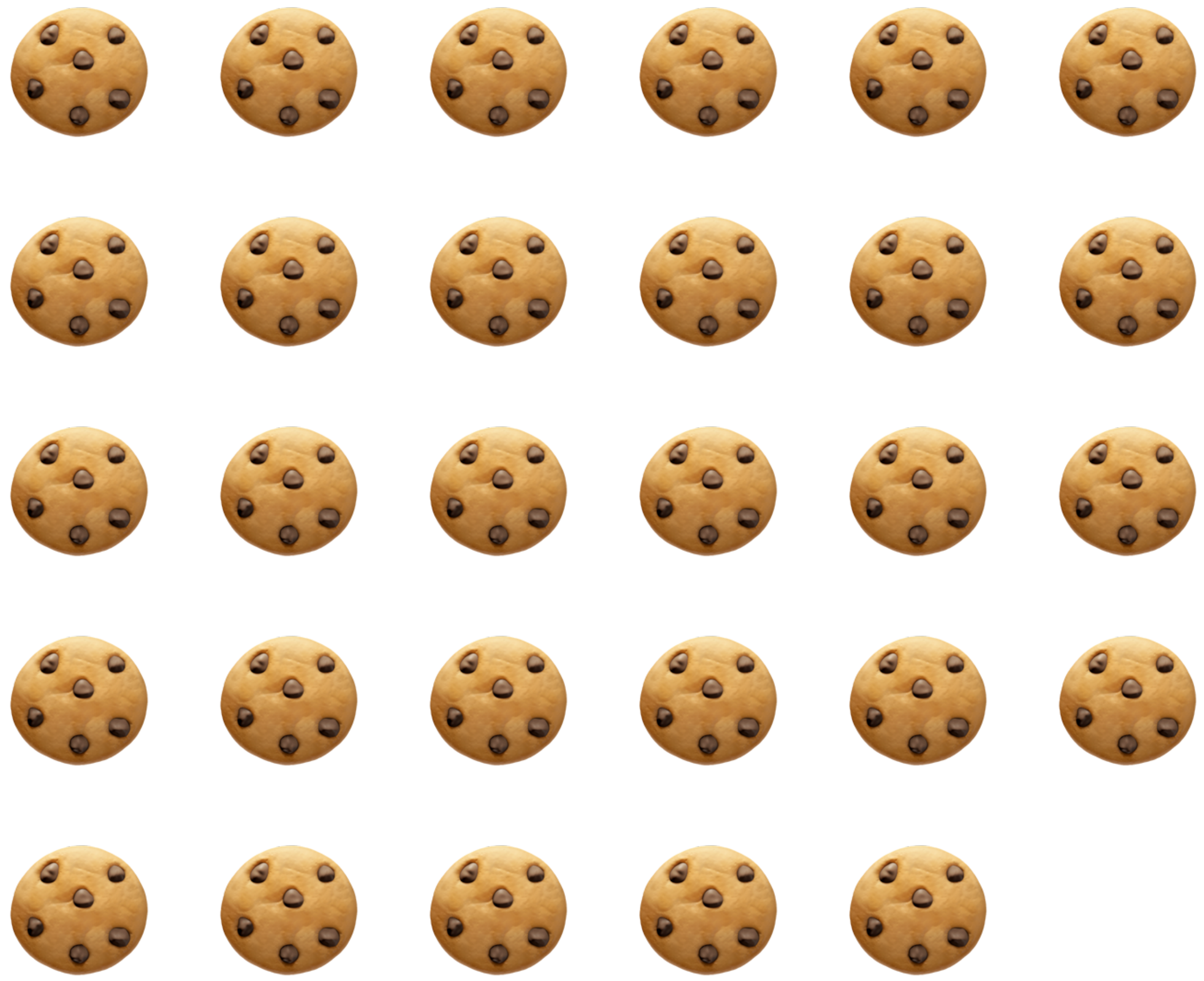


Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

- 2nd player cannot win no matter what she tries.

  So 1st player has a winning strategy.



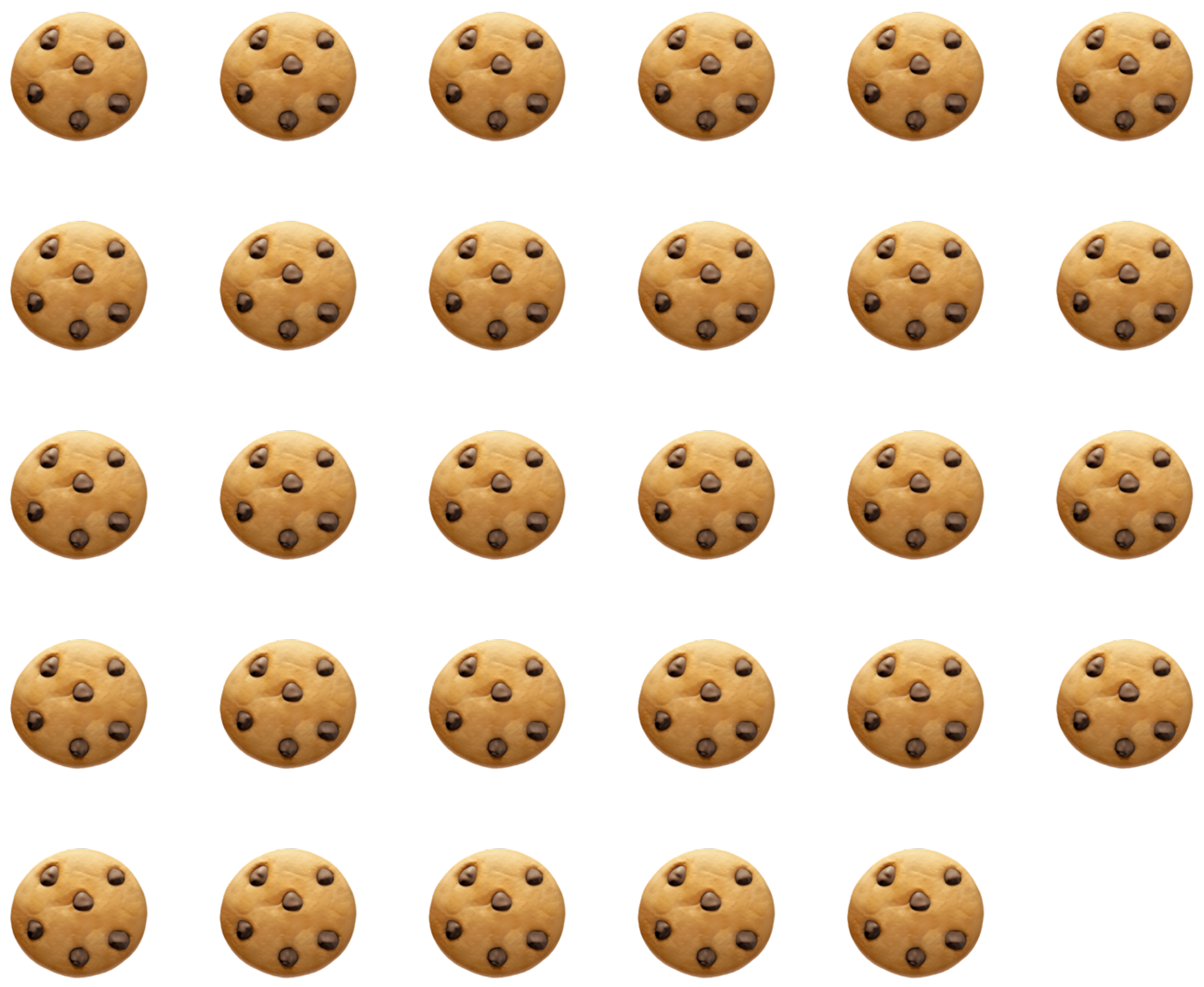
Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

- 2nd player cannot win no matter what she tries.

  So 1st player has a winning strategy.

- 2nd player can win by picking (i, j)th cookie.



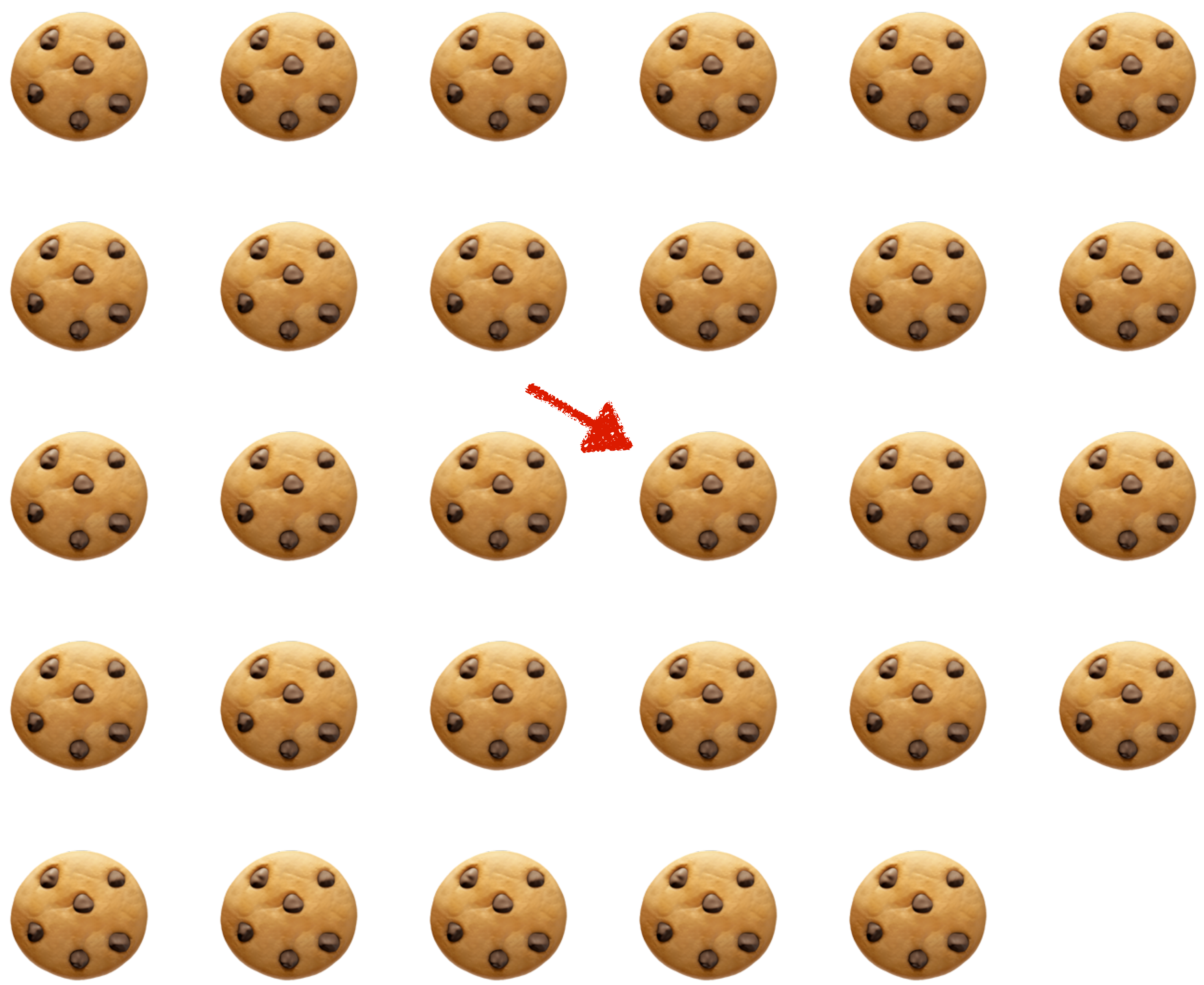
Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

- 2nd player cannot win no matter what she tries.

  So 1st player has a winning strategy.

- 2nd player can win by picking (i, j)th cookie.



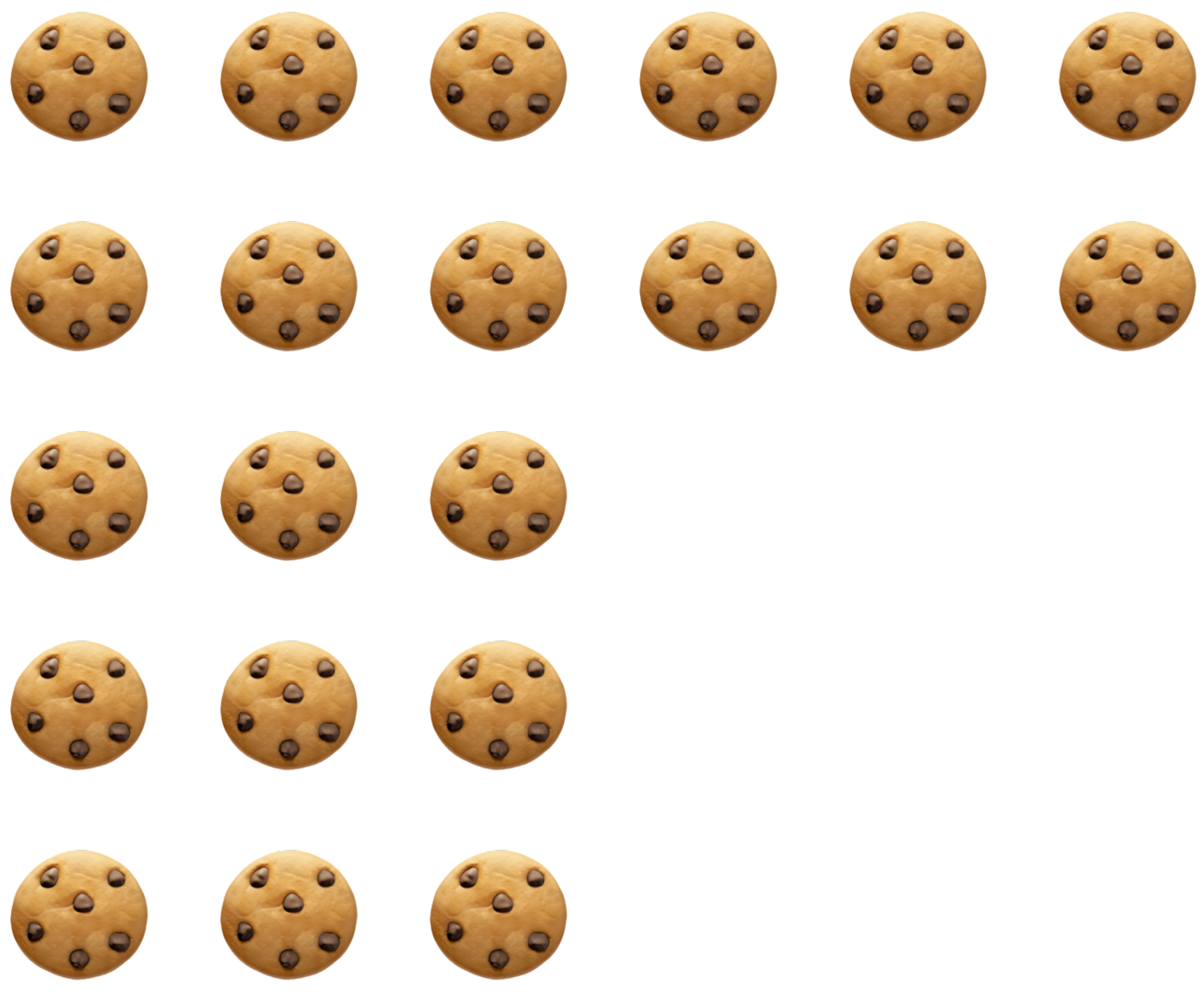
Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

- 2nd player cannot win no matter what she tries.

  So 1st player has a winning strategy.

- 2nd player can win by picking (i, j)th cookie.



Game of Chomp

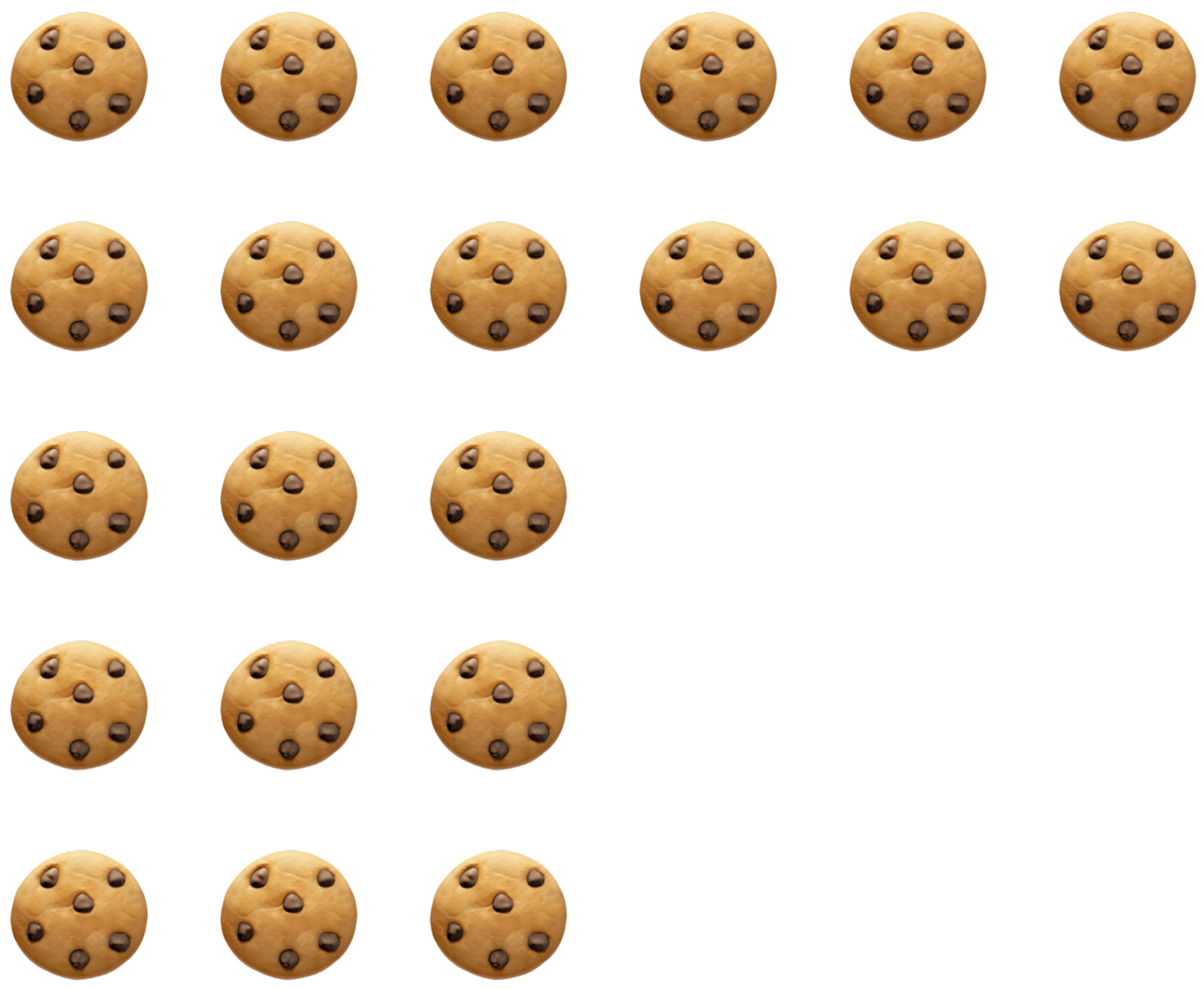
If 1st player eats the bottom-right cookie, two cases are possible:

- 2nd player cannot win no matter what she tries.

  So 1st player has a winning strategy.

- 2nd player can win by picking (i, j)th cookie.

  But then 1st player could have picked the (i, j)th cookie



Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

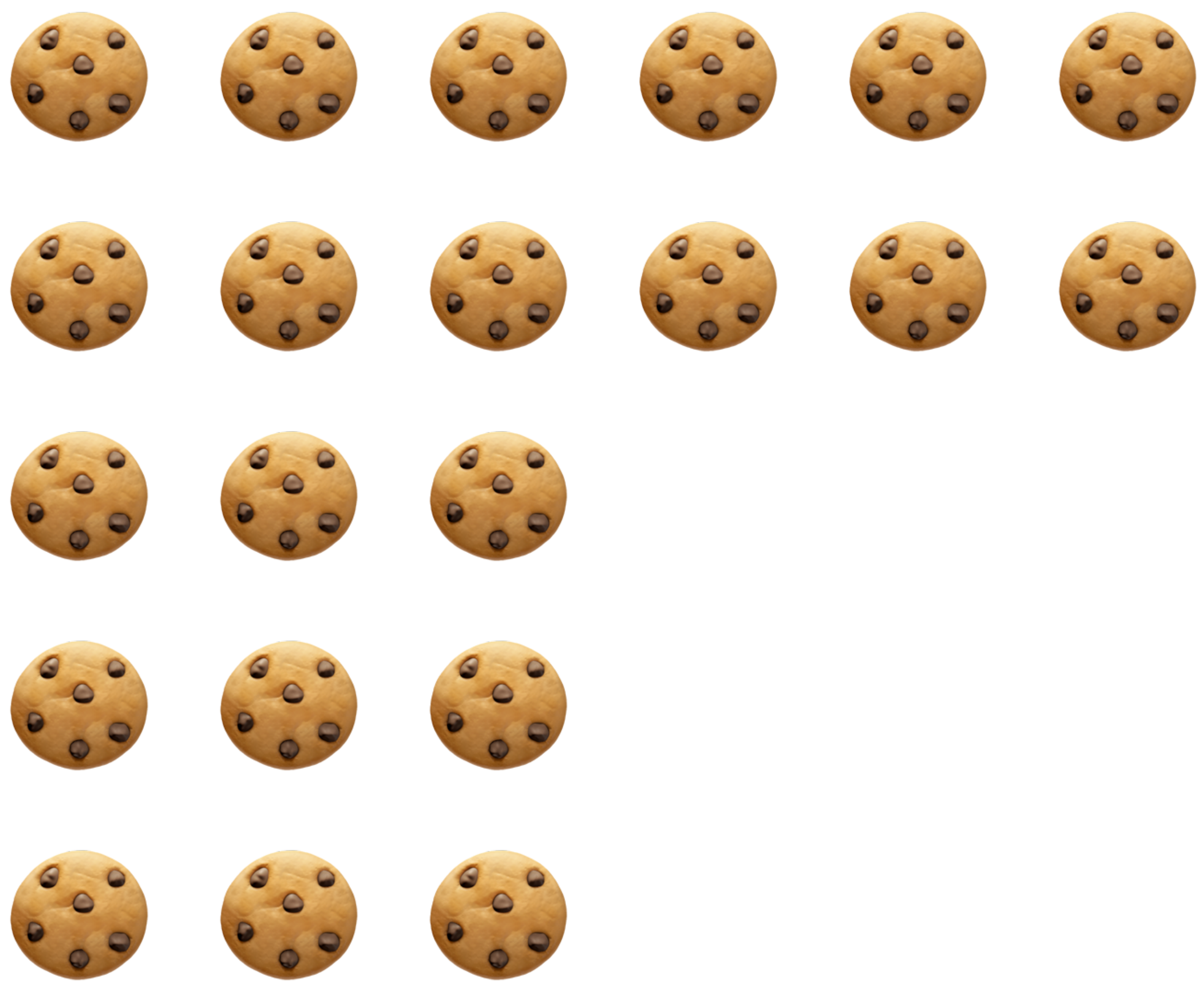
- 2nd player cannot win no matter what she tries.

  So 1st player has a winning strategy.

- 2nd player can win by picking (i, j)th cookie.

  But then 1st player could have picked the (i, j)th cookie

  on her 1st turn and forced 2nd player to lose.



Game of Chomp

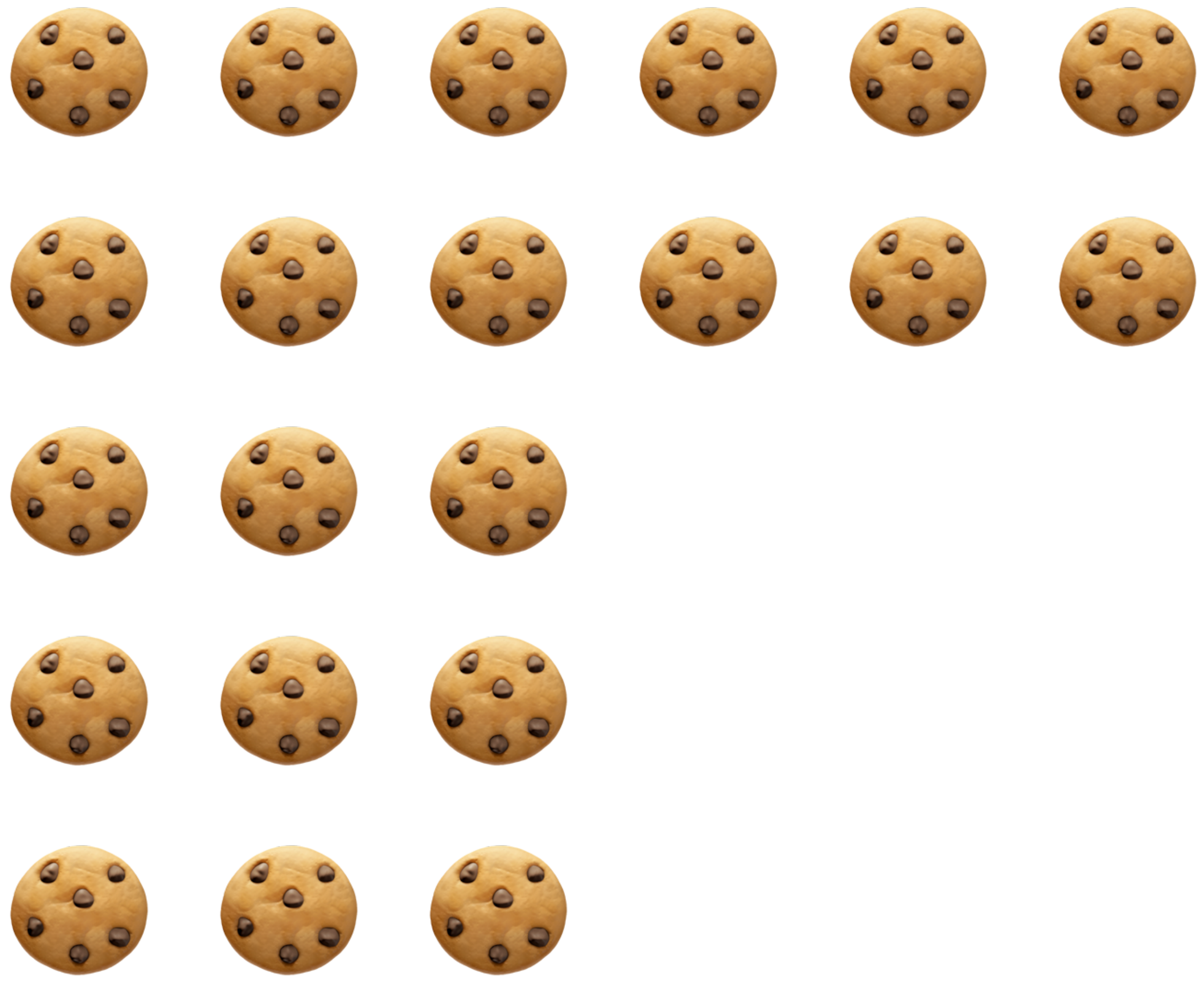
If 1st player eats the bottom-right cookie, two cases are possible:

- 2nd player cannot win no matter what she tries.

  So 1st player has a winning strategy.

- 2nd player can win by picking (i, j)th cookie.

  But then 1st player could have picked the (i, j)th cookie on her 1st turn and forced 2nd player to lose.

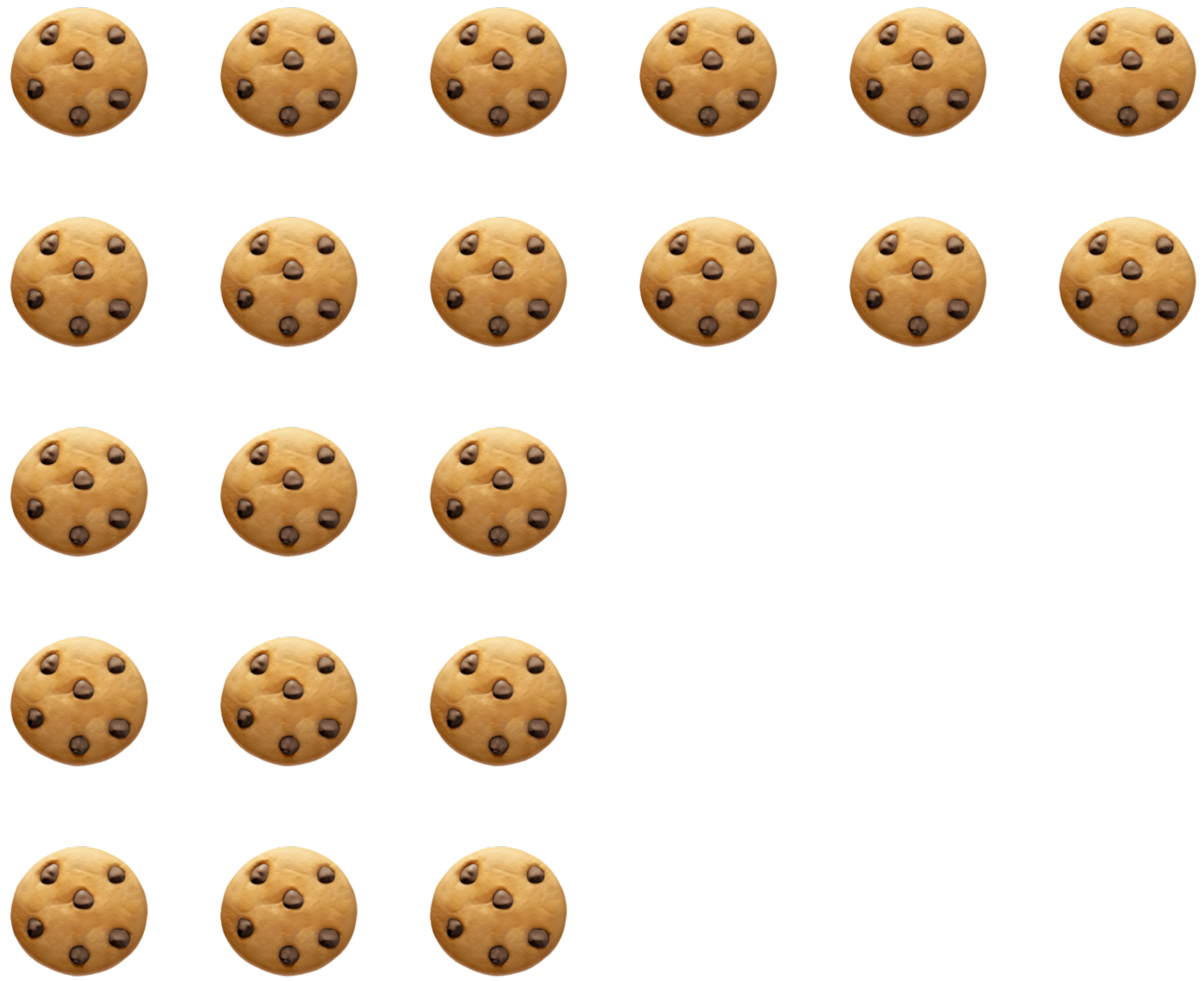


Note: We haven't shown that 1st player knows the

Game of Chomp

If 1st player eats the bottom-right cookie, two cases are possible:

- 2nd player cannot win no matter what she tries.
  So 1st player has a winning strategy.
- 2nd player can win by picking (i, j)th cookie.
  But then 1st player could have picked the (i, j)th cookie on her 1st turn and forced 2nd player to lose.



Note: We haven't shown that 1st player knows the winning strategy, we have only shown that it exists for her.

Forward and Backward Reasoning

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

p

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \ \rightarrow$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \;\rightarrow\; q_1 \;\rightarrow$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2 \rightarrow$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2 \rightarrow q_3$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow \cdots$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \to q_1 \to q_2 \to q_3 \to \cdots \to$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow \cdots \rightarrow q$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \to q_1 \to q_2 \to q_3 \to \cdots \to q$$

q is a known true proposition

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow \cdots \leftrightarrows q \longleftarrow$$

q is a known true proposition

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2 \rightarrow q_3 \leftrightarrows \cdots \leftrightarrows q \longleftarrow$$

q is a known true proposition

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \rightarrow q_2 \leftrightarrows q_3 \leftrightarrows \cdots \leftrightarrows q \longleftarrow \text{q is a known true proposition}$$

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \rightarrow q_1 \leftrightarrows q_2 \leftrightarrows q_3 \leftrightarrows \cdots \leftrightarrows q$$

q is a known true proposition

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \leftrightarrows q_1 \leftrightarrows q_2 \leftrightarrows q_3 \leftrightarrows \cdots \leftrightarrows q \longleftarrow$$

q is a known true proposition

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \leftrightarrows q_1 \leftrightarrows q_2 \leftrightarrows q_3 \leftrightarrows \cdots \leftrightarrows q$$

q is a known true proposition

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \leftrightarrows q_1 \leftrightarrows q_2 \leftrightarrows q_3 \leftrightarrows \cdots \leftrightarrows q$$

q is a known true proposition

Forward and Backward Reasoning

Two strategies to prove a mathematical statement, say p:

Forward Reasoning:

Try to prove p using premises, axioms, and existing theorems in a straightforward manner.

Backward Reasoning:

Assume p and try to deduce a true statement q in a way that you can also deduce p from q.

$$p \leftrightarrows q_1 \leftrightarrows q_2 \leftrightarrows q_3 \leftrightarrows \cdots \leftrightarrows q$$

q is a known true proposition

Deduction can happen in both the directions.

Example: Backward Reasoning

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$
- Geometric mean $= \sqrt{xy}$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

$$(x+y)/2 \geq \sqrt{xy}$$

- Arithmetic mean $= \dfrac{x+y}{2}$

- Geometric mean $= \sqrt{xy}$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$
- Geometric mean $= \sqrt{xy}$

$$\begin{aligned} (x+y)/2 &\geq \sqrt{xy} \\ (x+y)^2/4 &\geq xy \end{aligned}$$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$

- Geometric mean $= \sqrt{xy}$

$$\begin{aligned}(x+y)/2 &\geq \sqrt{xy}\\ (x+y)^2/4 &\geq xy\\ (x+y)^2 &\geq 4xy\end{aligned}$$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$
- Geometric mean $= \sqrt{xy}$

$$
\begin{aligned}
(x+y)/2 &\geq \sqrt{xy} \\
(x+y)^2/4 &\geq xy \\
(x+y)^2 &\geq 4xy \\
x^2+y^2+2xy &\geq 4xy
\end{aligned}
$$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$
- Geometric mean $= \sqrt{xy}$

$$
\begin{aligned}
(x+y)/2 &\geq \sqrt{xy} \\
(x+y)^2/4 &\geq xy \\
(x+y)^2 &\geq 4xy \\
x^2+y^2+2xy &\geq 4xy \\
x^2+y^2-2xy &\geq 0
\end{aligned}
$$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$
- Geometric mean $= \sqrt{xy}$

$$
\begin{aligned}
(x+y)/2 &\geq \sqrt{xy} \\
(x+y)^2/4 &\geq xy \\
(x+y)^2 &\geq 4xy \\
x^2+y^2+2xy &\geq 4xy \\
x^2+y^2-2xy &\geq 0 \\
(x-y)^2 &\geq 0
\end{aligned}
$$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$
- Geometric mean $= \sqrt{xy}$

Order of Reasoning

$$
\begin{aligned}
(x+y)/2 &\geq \sqrt{xy} \\
(x+y)^2/4 &\geq xy \\
(x+y)^2 &\geq 4xy \\
x^2+y^2+2xy &\geq 4xy \\
x^2+y^2-2xy &\geq 0 \\
(x-y)^2 &\geq 0
\end{aligned}
$$

Example: Backward Reasoning

Theorem: For any two positive real numbers x and y, their arithmetic mean is greater than or equal to their geometric mean.

Reasoning:

- Arithmetic mean $= \dfrac{x+y}{2}$
- Geometric mean $= \sqrt{xy}$

Order of Reasoning

$$
\begin{aligned}
(x+y)/2 &\geq \sqrt{xy} \\
(x+y)^2/4 &\geq xy \\
(x+y)^2 &\geq 4xy \\
x^2+y^2+2xy &\geq 4xy \\
x^2+y^2-2xy &\geq 0 \\
(x-y)^2 &\geq 0
\end{aligned}
$$

Proof should be in this order

Disproving Mathematical Statements

Disproving Mathematical Statements

How to disprove a mathematical statement, say p?

Disproving Mathematical Statements

How to disprove a mathematical statement, say p?

Prove $\neg p$.

Disproving Mathematical Statements

How to disprove a mathematical statement, say p?

Prove $\neg p$. (Be careful while forming $\neg p$.)